

Find the stationary path 1

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Problem 6.11 in Taylor

Find the path $y(x)$ for which the integral below is stationary

$$\int_{x_1}^{x_2} \underbrace{\sqrt{x} \sqrt{1 + (y')^2}}_{f(y', x)} dx$$

$$\frac{\partial f}{\partial y} = 0$$

$$\frac{\partial f}{\partial y'} = \frac{\sqrt{x} y'}{\sqrt{1 + (y')^2}}$$

$$\frac{d}{dx} \frac{\partial f}{\partial y'} = 0$$

$$\frac{\sqrt{x} y'}{\sqrt{1 + (y')^2}} = c \quad x (y')^2 = c^2 (1 + (y')^2)$$

$$(x - c^2)(y')^2 = c^2$$

$$y' = \frac{c}{\sqrt{x - c^2}}$$

$$dy = \frac{c dx}{\sqrt{x - c^2}}$$

$$y = 2c^2 \sqrt{\frac{x}{c^2} - 1} + d \quad c, d \text{ constants}$$

$$y(x_1) = y_1$$

$$y_1 = 2c^2 \sqrt{\frac{x_1}{c^2} - 1} + d$$

$$y(x_2) = y_2$$

$$y_2 = 2c^2 \sqrt{\frac{x_2}{c^2} - 1} + d$$

by solving these eqs one can in principle fix $c(x_1, y_1, x_2, y_2)$
 $d(x_1, y_1, x_2, y_2)$