Two body problem momenta

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Problem 8.5 in Taylor's book.

Find the momentum conjugate to the relative position in the two body problem. Find the relation between this momentum and the momenta of the two particles in the COM frame.

$$\mathcal{L} = \frac{1}{2} M \dot{\bar{R}}^2 + \frac{1}{2} \mu \dot{\bar{r}}^2 - U(r)$$

$$\bar{r} \equiv x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = \mu \dot{x} \qquad p_y = \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu \dot{y} \qquad p_z = \frac{\partial \mathcal{L}}{\partial \dot{z}} = \mu \dot{z}$$

$$\boxed{\bar{p} = \mu \dot{\bar{r}}}$$

rem $\mu = \dfrac{m_1 m_2}{M}$ $\bar{r} \equiv \bar{r}_1 - \bar{r}_2$

and in COM $m_1 \bar{r}_1 = -m_2 \bar{r}_2$

↳ $$\bar{r} = \bar{r}_1 - \bar{r}_2 = \bar{r}_1 + \frac{m_1}{m_2}\bar{r}_1 = \frac{M}{m_2}\bar{r}_1$$

↳ $$\bar{p} = \mu \dot{\bar{r}} = \frac{m_1 m_2}{M}\frac{M}{m_2}\dot{\bar{r}}_1 = m_1 \dot{\bar{r}}_1 = \bar{p}_1$$

similarly

$$\bar{r} = \bar{r}_1 - \bar{r}_2 = -\frac{m_2}{m_1}\bar{r}_2 - \bar{r}_2 = -\frac{m_1 + m_2}{m_1}\bar{r}_2 = -\frac{M}{m_1}\bar{r}_2$$

$$\bar{p} = \mu \dot{\bar{r}} = -\frac{m_1 m_2}{M}\frac{M}{m_1}\dot{\bar{r}}_2 = -m_2 \dot{\bar{r}}_2 = -\bar{p}_2$$