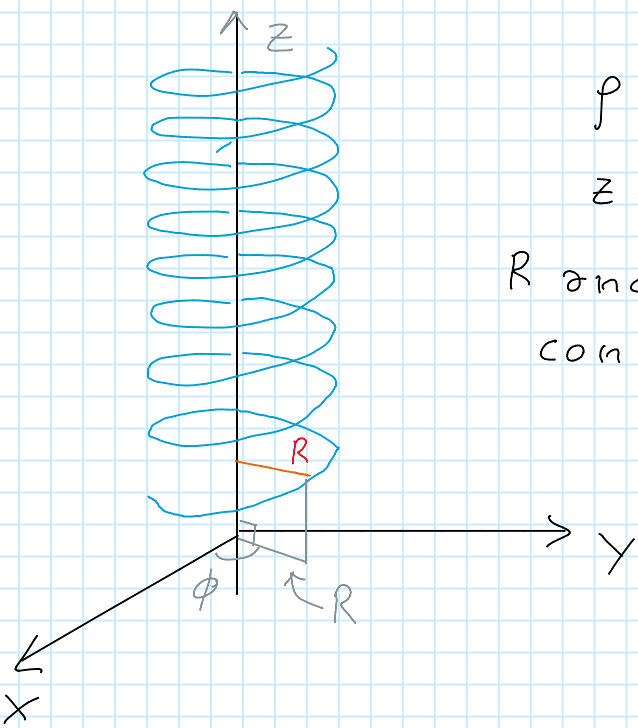
Lagrangian or the particle on a helix

Monday, November 2, 2020 7:42 AM

Problem 7.20 in Taylor.

A smooth wire is bent into the shape of an helix. Assume that the z axis is the axis of the cylinder around which the helix is wrapped. Write the Lagrangian for a bead threaded on the wire, using z as the generalized coordinate. Find the bead's acceleration along z. What happens to this accelerations when the radius of the cylinder goes to zero?



$\rho = R$

$z = \lambda \phi$

R and λ are constants

$$x = R\cos\phi = R\cos\frac{z}{\lambda} \qquad \dot{x} = -\frac{R}{\lambda}\dot{z}\sin\frac{z}{\lambda}$$

$$y = R\sin\phi = R\sin\frac{z}{\lambda} \qquad \dot{y} = +\frac{R}{\lambda}\dot{z}\cos\frac{z}{\lambda}$$

$$T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) = \frac{m}{2}\dot{z}^2\left(1 + \frac{R^2}{\lambda^2}\right)$$

$$U = mgz$$

$$\mathcal{L} = \frac{m\dot{z}^2}{2}\left(1 + \frac{R^2}{\lambda^2}\right) - mgz$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{z}} - \frac{\partial \mathcal{L}}{\partial z} = m\ddot{z}\left(1+\frac{R^2}{\lambda^2}\right) + mg = 0$$

$$\boxed{\ddot{z} = \frac{-g}{1+\frac{R^2}{\lambda^2}}}$$

if $R \to 0 \qquad \ddot{z} = -g$ ✓