Reconstruct amplitude and frequency

Wednesday, October 7, 2020 9:26 AM

Problem 5.11 in Taylor

Assume to know the position and velocity of an oscillating mass at two moments in time. What can be said about the amplitude and the frequency of the oscillation?

at t_1 $\quad x(t_1) \equiv x_1$, $\quad v(t_1) \equiv v_1$

at t_2 $\quad x(t_2) \equiv x_2$, $\quad v(t_2) \equiv v_2$

$$x(t) = A\cos(\omega t - \delta)$$

$$v(t) = -A\omega\sin(\omega t - \delta)$$

$$x_1 = A\cos(\omega t_1 - \delta) \qquad v_1 = -A\omega\sin(\omega t_1 - \delta)$$

$$x_2 = A\cos(\omega t_2 - \delta) \qquad v_2 = -A\omega\sin(\omega t_2 - \delta)$$

$$x_1^2 + \frac{v_1^2}{\omega^2} = A^2 \qquad x_2^2 + \frac{v_2^2}{\omega^2} = A^2$$

$$x_1^2 + \frac{v_1^2}{\omega^2} = x_2^2 + \frac{v_2^2}{\omega^2} \longrightarrow x_1^2 - x_2^2 = \frac{v_2^2 - v_1^2}{\omega^2}$$

$$\boxed{\omega = \sqrt{\frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}}}$$

$$x_1^2 + \frac{v_1^2}{\omega^2} = A^2 \qquad x_1^2 + \frac{v_1^2}{v_2^2 - v_1^2}(x_1^2 - x_2^2) = A^2$$

$$x_1^2\,\frac{v_2^2 - \cancel{v_1^2} + \cancel{v_1^2}}{v_2^2 - v_1^2} - \frac{v_1^2 x_2^2}{v_2^2 - v_1^2} = A^2$$

$$\frac{x_1^2 v_2^2 - v_1^2 x_2^2}{v_2^2 - v_1^2} = A^2$$

$$\boxed{A = \sqrt{\frac{x_1^2 v_2^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}}}$$