Oscillations

Tuesday, October 6, 2020 9:05 AM

Problem 5.10 in Taylor.

An object that can move on the x axis is subject to a force

$$F = -F_0 \sinh(\alpha x) \qquad F_0, \alpha > 0 \qquad [\alpha] = \frac{1}{m}$$

Find the potential U

$$U(x) = -\int_{x_0}^{x} F(x)\,dx = F_0 \int_{x_0}^{x} \frac{e^{\alpha x} - e^{-\alpha x}}{2}\,dx$$

$$= \frac{F_0}{2\alpha}\left\{\left[e^{\alpha x}\right]_{x_0}^{x} + \left[e^{-\alpha x}\right]_{x_0}^{x}\right\}$$

$$= \frac{F_0}{\alpha}\left[\cosh(\alpha x)\right]_{x_0}^{x} = \frac{F_0}{\alpha}\cosh(\alpha x) - \frac{F_0}{\alpha}\overbrace{\cosh(\alpha x_0)}^{=1 \text{ if } x_0 = 0}$$

The potential has a minimum near x = 0 (where the force vanishes) find the angular frequency of the oscillations around the minimum.

$$U(x) = U(0) + \underbrace{\cancelto{0}{\left.\frac{dU}{dx}\right|_{x=0}}}_{\text{at minimum}} x + \frac{1}{2}\left.\frac{d^2U}{dx^2}\right|_{x=0} x^2 + \dots$$

$$U(0) = \frac{F_0}{\alpha}\cos(\alpha 0) - \frac{F_0}{\alpha} = 0$$

$$\frac{d^2U}{dx^2} = \frac{d}{dx} F_0 \sinh(\alpha x) = F_0 \alpha \cosh(\alpha x)$$

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = F_0 \alpha$$

$$U(x) \simeq + \frac{1}{2} F_0 \alpha x^2$$

Therefore

$$m\ddot{x} = -\frac{dU}{dx} \simeq -F_0 \alpha x \longrightarrow \omega^2 = \frac{F_0 \alpha}{m}$$

$$\left[\frac{F_0 \alpha}{m}\right] = \frac{\cancel{kg}\frac{m}{s^2}}{\cancel{kg}} \frac{1}{m} = \frac{1}{s^2} \quad \checkmark$$