Separation of variables, v vs t

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Problem 2.7

Assume that the force applied to an object that can move in one dimension depends only on the velocity

$$m\dot{v} = F(v) \qquad \frac{dv}{dt} = \frac{F(v)}{m}$$

$$dt = \frac{m}{F(v)}\,dv$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{m}{F(v)}\,dv \qquad t = m\int_{v_0}^{v}\frac{dv}{F(v)}$$

The simplest case is the case in which the force (and consequently the acceleration) are constant

$$t = \frac{m}{F}\int_{v_0}^{v} dv = \frac{m}{F}(v - v_0)$$

$$v = v_0 + \underbrace{\frac{F}{m}}_{=a}\, t$$

Problem 2.8

Assume $F(v) = -c\,v^{\frac{3}{2}}$

and $v(t=0) = v_0$

$$t = -m\int_{v_0}^{v}\frac{dv}{c\,v^{\frac{3}{2}}} = -\frac{m}{c}\int_{v_0}^{v}\frac{dv}{v^{\frac{3}{2}}} = +\frac{2m}{c}\left[\frac{1}{\sqrt{v}}\right]_{v_0}^{v}$$

$$= \frac{2m}{c}\left[\frac{1}{\sqrt{v}} - \frac{1}{\sqrt{v_0}}\right]$$

$$\frac{ct}{2m} + \frac{1}{\sqrt{v_0}} = \frac{1}{\sqrt{v}} \qquad \frac{\sqrt{v_0}\,ct + 2m}{2m\sqrt{v_0}} = \frac{1}{\sqrt{v}}$$

$$v = \frac{4m^2 v_0}{\left(2m + \sqrt{v_0}\,ct\right)^2}$$