Linear drag force

Tuesday, September 15, 2020 3:33 AM

problem 2.6

Start from equation 2.33

$$v_y = v_{ter}\left(1 - e^{-\frac{t}{\tau}}\right) \qquad \tau \equiv \frac{m}{b} \qquad v_{ter} = \frac{mg}{b}$$

for $t \ll \tau$

$$e^{-\frac{t}{\tau}} = 1 - \frac{t}{\tau} + \frac{1}{2}\frac{t^2}{\tau^2} - \frac{1}{6}\frac{t^3}{\tau^3} + \dots$$

therefore

$$v_y = v_{ter}\left(1 - 1 + \frac{t}{\tau} + \dots\right)$$

$$= \frac{v_{ter}}{\tau} t = \frac{mg}{b}\frac{b}{m} t = gt \quad ✓$$

one can then expand the equation for y(t)

$$y(t) = v_{ter}t + (v_{0,y} - v_{ter})\tau\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$= v_{ter}t + (v_{0,y} - v_{ter})\tau\left(1 - 1 + \frac{t}{\tau} - \frac{1}{2}\frac{t^2}{\tau^2} + \dots\right)$$

$$= v_{ter}t + (v_{0,y} - v_{ter})\left(\cancel{\tau}\frac{t}{\cancel{\tau}} - \frac{1}{2}\cancel{\tau}\frac{t^2}{\tau^{\cancel{2}}} + \dots\right)$$

$$= \cancel{v_{ter}t} + v_{0,y}t - \cancel{v_{ter}t} - \frac{1}{2}v_{0,y}\frac{t^2}{\tau} + \frac{1}{2}v_{ter}\frac{t^2}{\tau} + \dots$$

$$= v_{0,y}t + \frac{1}{2}\underbrace{\frac{mg}{b}}_{v_{ter}}\underbrace{\frac{b}{m}}_{\frac{1}{\tau}}t^2 - \frac{1}{2}v_{0,y}t\underbrace{\frac{t}{\tau}}_{\text{supressed}} + \dots$$

$$= v_{0,y}t + \frac{1}{2}gt^2 + \dots \quad ✓$$