Hamiltonian of a particle in free fall

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Taylor, problem 13.2

Consider a mass m constrained to move in a vertical line under the influence of gravity. Using the coordinate x measured vertically down from a convenient origin O, write down the Lagrangian and find the generalized momentum. Find the Hamiltonian as a function of x and p, and write down Hamilton's equations of motion.

$$\mathcal{L} = \frac{1}{2} m \dot{x}^2 + mgx$$

rem x is pointing downward, as stated in the text of the problem

$$U = -mgx$$

$$p = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x} \longrightarrow \dot{x} = \frac{p}{m}$$

$$H = p\dot{x} - \mathcal{L}(x, \dot{x}(x,p))$$

$$= \frac{p^2}{m} - \frac{p^2}{2m} - mgx = \frac{p^2}{2m} - mgx$$

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m} \qquad \dot{p} = -\frac{\partial H}{\partial x} = mg$$