Pendulum in an accelerating elevator

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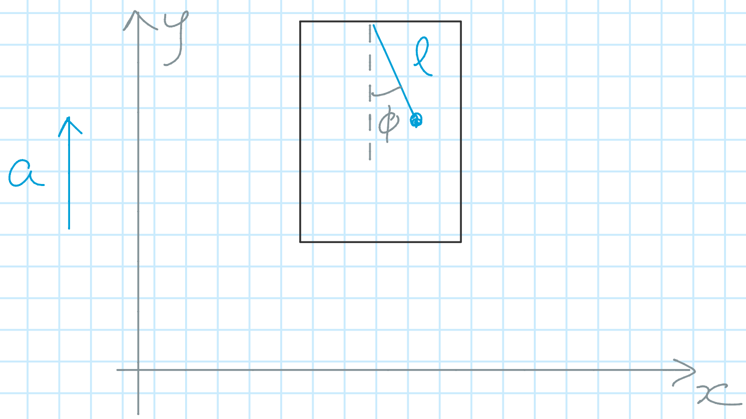
Problem 7.22 in Taylor's book
Using the angle that the pendulum make with the vertical direction as the generalized coordinate, write down the Lagrangian of the simple pendulum of length l suspended from the ceiling of an elevator that is accelerating upward with constant acceleration a. Show that the Lagrange equation of motion is the same as for the regular pendulum, except that g has been replaced by g + a.



$$x = l\sin\phi \qquad y = \frac{1}{2}at^2 - l\cos\phi + y_0$$

$$\dot{x} = \dot{\phi}\, l\cos\phi \qquad \dot{y} = at + \dot{\phi}\, l\sin\phi$$

$$\dot{x}^2 + \dot{y}^2 = l^2\dot{\phi}^2 + a^2t^2 + 2a\dot{\phi}\, l t\sin\phi$$

$$\mathcal{L} = \frac{m}{2}\left(l^2\dot{\phi}^2 + a^2t^2 + 2a\dot{\phi}\, l t\sin\phi\right) - mg\left(\frac{1}{2}at^2 - l\cos\phi\right)$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\phi}} - \frac{\partial\mathcal{L}}{\partial\phi} = ml^2\ddot{\phi} + mal\left(\sin\phi + \cancel{t\dot{\phi}\cos\phi}\right) - mal\cancel{t\dot{\phi}\cos\phi}$$

$$+ mgl\sin\phi = 0$$

$$\boxed{l\ddot{\phi} + a\sin\phi + g\sin\phi = 0}$$

$$\ddot{\phi} = -\frac{g+a}{l}\sin\phi$$