Orbital period

Sunday, October 6, 2019 11:46 AM

One can then easily find the orbital period of the elliptic motion in a bound Kepler orbit. From Kepler second law (which is a consequence of the conservation of angular momentum) one has that

$$\frac{dA}{dt} = \frac{\ell}{2\mu}$$

The area of the ellipse is

$$A = \pi a b$$

Therefore the period of the motion is

$$\tau = \frac{A}{\frac{dA}{dt}} = \pi a b \, \frac{2\mu}{\ell}$$

$$\tau^2 = \frac{4\pi^2\mu^2}{\ell^2} a^2 b^2 = \frac{4\pi^2\mu^2}{\ell^2} \underbrace{\frac{c^2}{(1-\epsilon^2)^2}}_{a^2} \underbrace{\frac{c^2}{1-\epsilon^2}}_{b^2}$$

$$= \frac{4\pi^2\mu^2}{\ell^2} \underbrace{\frac{c^4}{(1-\epsilon^2)^3}}_{a^3 c} = \frac{4\pi^2\mu^2}{\ell^2} \underbrace{c}_{=\frac{\ell^2}{\gamma\mu}} a^3$$

$$\boxed{\tau^2 = 4\pi^2 \frac{\mu}{\gamma} a^3}$$

Finally, remember that

$$\gamma = G m_1 m_2 \simeq G \mu M$$

mass of the heavy object

example sun in planetary motion

KEPLER THIRD LAW

$\tau^2 \propto a^3$

$$\boxed{\tau^2 = \frac{4\pi^2}{GM} a^3}$$