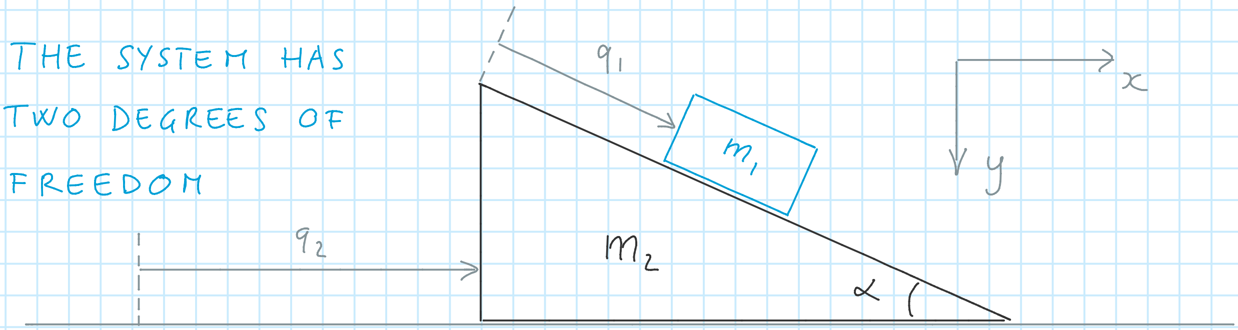
Block sliding on a sliding wedge

Monday, September 2, 2019 11:09 AM

In this example we consider a block sliding without friction. However, we also assume that the wedge can slide without friction on the table on which it is resting. In addition, we want to figure out how long it takes to the block to reach the table. This is a case in which the equations of motion are more easily written down by using the Lagrangian formalism.

THE SYSTEM HAS TWO DEGREES OF FREEDOM



$$\bar{v}_2 = \dot{q}_2 \hat{\imath} \qquad \bar{v}_1 = (\dot{q}_1 \cos\alpha + \dot{q}_2)\hat{\imath} + \dot{q}_1 \sin\alpha \hat{\jmath}$$

$$T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 (\dot{q}_1^2 + \dot{q}_2^2 + 2\dot{q}_1\dot{q}_2 \cos\alpha) + \frac{1}{2} m_2 \dot{q}_2^2$$

$$U = -m_1 g q_1 \sin\alpha$$

$$\mathcal{L} = \frac{1}{2}(m_1 + m_2)\dot{q}_2^2 + \frac{1}{2} m_1 (\dot{q}_1^2 + 2\dot{q}_1\dot{q}_2 \cos\alpha) + m_1 g q_1 \sin\alpha$$

One can then write down Lagrange's equations

$$\frac{\partial \mathcal{L}}{\partial q_2} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_2} = -\frac{d}{dt}\left[(m_1+m_2)\dot{q}_2 + m_1 \dot{q}_1 \cos\alpha\right] = 0$$

$$\boxed{\underbrace{m_2\dot{q}_2 + m_1(\dot{q}_2 + \dot{q}_1\cos\alpha) = \text{const}}_{\text{conservation of momentum along } x}}$$

$$\frac{\partial \mathcal{L}}{\partial q_1} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = m_1 g\sin\alpha - m_1(\ddot{q}_1 + \ddot{q}_2\cos\alpha) = 0$$

It is convenient to take the derivative of the equation for the conservation of the momentum along x so that one finds a relation between the two accelerations that can subsequently be plugged into the Lagrange's equation for q_1

$$m_2\ddot{q}_2 + m_1\ddot{q}_2 + m_1\ddot{q}_1\cos\alpha = 0$$

$$\ddot{q}_2 = -\frac{m_1\ddot{q}_1\cos\alpha}{m_1+m_2}$$

$$m_1 g\sin\alpha - m_1\left(\ddot{q}_1 - \frac{m_1\ddot{q}_1\cos^2\alpha}{m_1+m_2}\right) = 0$$

$$g\sin\alpha - \ddot{q}_1\left(1 - \frac{m_1\cos^2\alpha}{m_1+m_2}\right) = 0$$

$$\boxed{\ddot{q}_1 = \frac{g\sin\alpha}{1 - \frac{m_1\cos^2\alpha}{m_1+m_2}}}$$

BLOCK'S ACCELERATION (it is constant)

observe that: $\begin{cases} \alpha = \pi/2 \rightarrow \ddot{q}_1 = g & \checkmark \\ m_2 \gg m_1 \rightarrow \ddot{q}_1 = g\sin\alpha & \checkmark \end{cases}$

The acceleration of the block along the wedge is constant. If the wedge has a length l, the time t that it takes to the block to slide all the way to the table will be

$$\ell = \frac{1}{2}\ddot{q}_1 t^2 \longrightarrow t = \sqrt{\frac{2\ddot{q}_1}{\ell}}$$