Position of a rocket in vertical motion

Tuesday, September 10, 2019 10:24 AM

From Taylor problem 3.13

In class we found out that the velocity of a rocket moving along the y axis and starting from rest is

$$v(t) = -gt + v_{ex} \ln\left(\frac{m_o}{m_o - kt}\right)$$

Find the position of the rocket along the y axis at a time t.

Solution

$$m(t) \equiv m_o - kt$$

$$\frac{dy}{dt} = -gt - v_{ex} \ln\left(\frac{m_o - kt}{m_o}\right)$$

$$dy = -\left[gt + v_{ex} \ln\left(\frac{m_o - kt}{m_o}\right)\right] dt$$

$$y - y_o = -\frac{1}{2} g t^2 - v_{ex} \int_0^t dt \ln\left(\frac{m_o - kt}{m_o}\right)$$

$$= -\frac{1}{2} g t^2 + v_{ex} \frac{m_o}{k} \int_1^u du \ln u$$

$$= -\frac{1}{2} g t^2 + v_{ex} \frac{m_o}{k} \left(u \ln u - u\right)\Big|_1^u$$

$$= -\frac{1}{2} g t^2 + v_{ex} \frac{m_o}{k} \left(\frac{m}{m_o} \ln \frac{m}{m_o} - \frac{m}{m_o} + 1\right)$$

$$= -\frac{1}{2} g t^2 + \frac{v_{ex} m_0}{k} \left(\frac{m_0 - m}{m_0} \right) + v_{ex} \frac{m}{k} \ln \frac{m}{m_0}$$

$$= -\frac{1}{2} g t^2 + v_{ex} t + v_{ex} \frac{m}{k} \ln \frac{m}{m_0}$$