# Eigenvalue equation

Wednesday, November 27, 2019 5:23 PM

A practical problem that it is often necessary to solve is the diagonalization of the inertia tensor. Once an origin of the frame of reference has been decided upon and the inertia tensor has been calculated with respect to a random right handed frame with origin in O, how can one find the principal axes of the problem and consequently find the diagonal form of the inertia tensor?

We are searching for the axes of rotation for which the angular momentum and the angular velocity point in the same direction

$$\bar{L} = \lambda \, \bar{\omega}$$

However, we know that on general grounds

$$\bar{L} = I \cdot \bar{\omega}$$

So we are actually interested in solving the equation

$$I \cdot \underbrace{\bar{\omega}}_{\text{eigenvector of the matrix } I} = \underbrace{\lambda}_{\text{eigenvalues of the matrix } I} \, \bar{\omega} \longrightarrow (I - \lambda \mathbb{1}) \cdot \bar{\omega} = 0$$

The goal is to find both the λ and the ω that satisfy the equations above. To start with the equations above have a solution if and only if

$$\boxed{\det(I - \lambda \mathbb{1}) = 0}$$

CHARACTERISTIC EQUATION (SECULAR EQUATION)

Since I is a 3x3 matrix, this is a cubic equation for the variable λ. Once the eigenvalues have been determined, it is necessary to find the eigenvectors by solving the equation

$$\boxed{(I - \lambda \mathbb{1}) \cdot \bar{\omega} = 0}$$