# PHYS 3100 - Final exam

## Problem 1

Consider a mass $m$ constrained to move in a vertical line under the influence of gravity (near the surface of the Earth).

a) Using the coordinate $y$ measured vertically down from a convenient origin O, write down the Lagrangian $\mathcal{L}$ and the generalized momentum $p = \partial\mathcal{L}/\partial\dot{y}$.*(5 points)*

b) Find the Hamiltonian $\mathcal{H}$ as a function of $y$ and $p$. *(10 points)*

c) Write down Hamilton's equations of motion. *(10 points)*

## Problem 2

A simple pendulum (mass $m$ length $l$) whose point of support $P$ is attached to the edge of a wheel (center $O$, radius $R$) that is forced to rotate counter-clockwise at a fixed angular velocity $\omega$. At the time $t = 0$, the point $P$ is level with $O$ on the right.

a) Write down the Lagrangian as a function of the angle between the a vertical line going through P and the pendulum. Indicate this angle with $\phi$. *(10 points)*

b) Find the equation of motion for the angle $\phi$. *(10 points)*

c) Check that your answer to part b) makes sense in the special case $\omega = 0$. Explain why your result makes sense. *(5 points)*

*Hint*: Be careful writing the kinetic energy $T$. A safe way to get to the velocity correctly is to write down the position of the bob at the time $t$, and then differentiate.

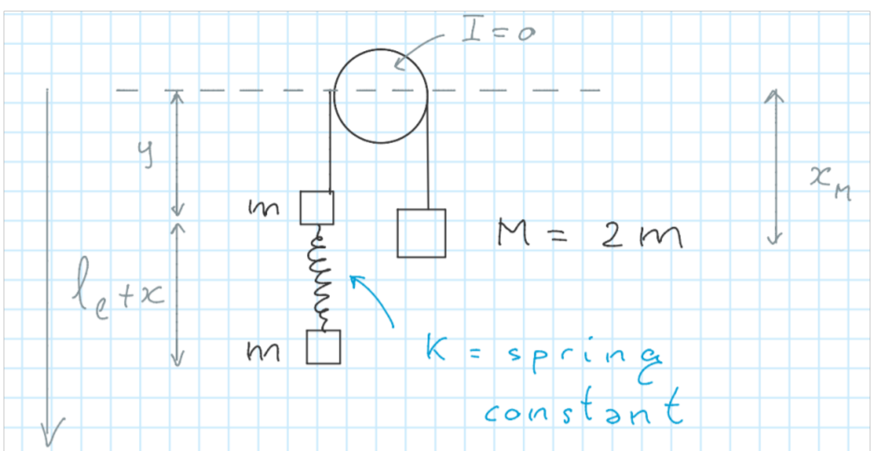
## Problem 3

Consider the modified Atwood machine shown in the figure. The two weights on the left have equal masses $m$ and are connected by a massless spring of force constant $k$. The weight on the right has a mass $M = 2m$, and the pulley is massless and frictionless. The coordinate $x$ is the extension of the spring from its equilibrium length; that is, the length of the spring is $l_e + x$ where $l_e$ is the equilibrium length (with all of the weights in position and $M$ held stationary).

a) Show that the total potential energy (spring plus gravitational) is just $U = kx^2/2$ (plus a constant that we can take to be zero). *(10 points)*

b) Find the two momenta conjugate to $x$ and $y$. Solve for $\dot{x}$ and $\dot{y}$, and write down the Hamiltonian. Show that coordinate $y$ is ignorable. *(10 points)*

c) Write down the four Hamilton equations and solve them for the following initial conditions: You hold the mass $M$ fixed with the whole system in equilibrium and $y = y_0$. Still holding $M$ fixed, you pull the lower mass $m$ down a distance $x_0$, and at $t = 0$ you let go of both masses. Describe the motion. In particular, find the frequency with which $x$ oscillates. *(5 points)*



*Hints*: Remember that the equilibrium length $l_e$ is not the same as the natural length $l$. To deal with the initial conditions, write down the initial values of $x$, $y$ and their momenta. You can solve the $x$ equations by combining them into a second order equation for $x$. Once you know $x(t)$, you can quickly write down the other three variables.

## Problem 4

A rigid body consists of three masses fastened as follows: $m$ at $(a, 0, 0)$, $2m$ at $(0, a, a)$ and $3m$ at $(0, a, -a)$.

a Find the inertia tensor $\mathbf{I}$. *(10 points)*

b Find the principal moments. *(5 points)*

b Find a set of orthogonal principal axes. *(10 points)*