Phase space of a repulsive force

Wednesday, November 20, 2019 8:45 AM

Taylor, problem 13.25

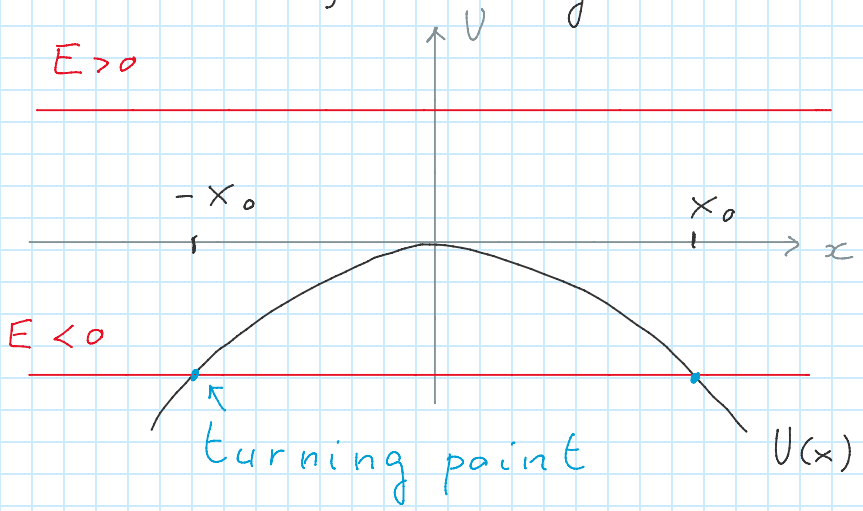
Consider a mass m confined to the x axis and subject to a force F = k x, where k > 0.

Write down the potential and describe the possible motions of the mass.

Write down the Hamiltonian and describe the possible phase space orbits.

$$F_x = -\frac{dU}{dx} = kx \qquad U = -\frac{1}{2}kx^2$$

(constant fixed by choosing $U(0) = 0$)



E = 0 $T = 0$ at $x = 0$

E > 0 $T > 0$ always, the object moves from left to right or from right to left

E < 0 $T > 0$ only for $x < -x_0$, $x > x_0$

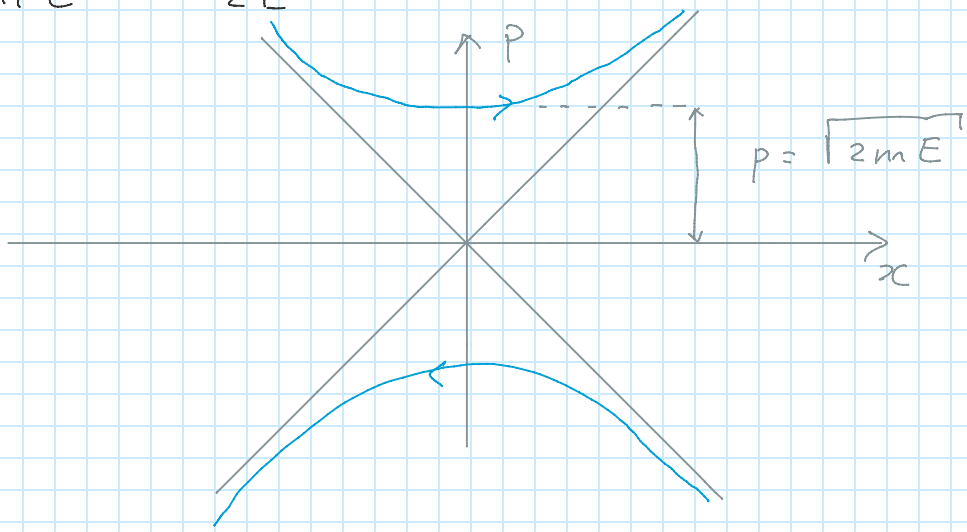
The region $-x_0 < x < x_0$ is not accessible

$$H(x,p) = \frac{p^2}{2m} - \frac{1}{2}kx^2 = E$$

for $E > 0$

$$\frac{p^2}{2mE} - \frac{kx^2}{2E} = 1$$

HYPERBOLA



for $E < 0$

$$-\frac{p^2}{2m|E|} + \frac{kx^2}{2|E|} = 1$$