Harmonic motion and constant force

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Taylor, problem 13.10

Consider a particle of mass m moving in two dimensions, subject to the force

$$\bar{F} = -kx\,\hat{i} + K\,\hat{j}$$

where k and K are positive constants.

Write down the Hamiltonian and Hamilton's equations, using x and y as generalized coordinates. Solve the equations and describe the motion.

One needs to find the potential first

$$U(x,y) = -\int \bar{F}\cdot d\bar{\ell} = -\int F_x\,dx - \int F_y\,dy + \text{const}$$

$$= \frac{kx^2}{2} - Ky + \text{const.}$$

$$\mathcal{H} = T + U = \frac{p_x^2 + p_y^2}{2m} + \frac{kx^2}{2} - Ky$$

$$\dot{x} = \frac{\partial \mathcal{H}}{\partial p_x} = \frac{p_x}{m} \qquad \dot{y} = \frac{\partial \mathcal{H}}{\partial p_y} = \frac{p_y}{m}$$

$$\dot{p}_x = -\frac{\partial \mathcal{H}}{\partial x} = -kx \qquad \dot{p}_y = -\frac{\partial \mathcal{H}}{\partial y} = K$$

$$\ddot{x} = -\frac{k}{m}x \qquad \ddot{y} = \frac{K}{m}$$

$$x = A\cos(\omega t + \delta)$$

SIMPLE HARMONIC MOTION

$$y = \frac{1}{2}\frac{K}{m}t^2 + v_{0,y}\,t + y_0$$

UNIFORMLY ACCELERATED MOTION