Particle on an helix

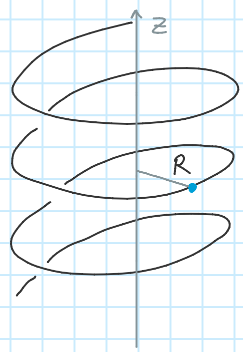
Wednesday, November 13, 2019 9:04 AM

Taylor, problem 13.5

A bead of mass m is threaded on a frictionless wire that is bent on an helix with cylindrical polar coordinates satisfying

$$z = c\phi \qquad \rho = R$$

With c and R constants. The z axis points vertically up and gravity points vertically down. Write down the Hamiltonian and Hamilton's equations. Find the angular acceleration and the acceleration along z.



$$\mathcal{L} = \frac{1}{2} m \left(R^2 \dot\phi^2 + \underbrace{c^2 \dot\phi^2}_{\dot z^2} \right) - mg \underbrace{c\phi}_{z}$$

$$p_\phi = \frac{\partial \mathcal{L}}{\partial \dot\phi} = m \left(R^2 + c^2 \right) \dot\phi \equiv p$$

$$\mathcal{H} = p\dot\phi - \mathcal{L} = \frac{p^2}{m(R^2+c^2)} - \frac{\cancel{m}}{2} \cancel{(R^2+c^2)} \frac{p^2}{m^{\cancel{2}}(R^2+c^2)^{\cancel{2}}}$$

$$+ mgc\phi$$

$$= \frac{p^2}{2m(R^2+c^2)} + mgc\phi$$

$$\dot{\phi} = \frac{\partial \mathcal{H}}{\partial p} = \frac{p}{m(R^2+c^2)}$$

$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial \phi} = -mgc$$

$$\dot{p} = \cancel{m}(R^2+c^2)\ddot{\phi} = -\cancel{m}gc$$

$$\ddot{\phi} = \frac{-gc}{R^2+c^2}$$

$$\ddot{z} = -\frac{gc^2}{R^2+c^2} \xrightarrow{R \to 0} \ddot{z} = -g \quad \checkmark$$