Mass hanging from a pulley

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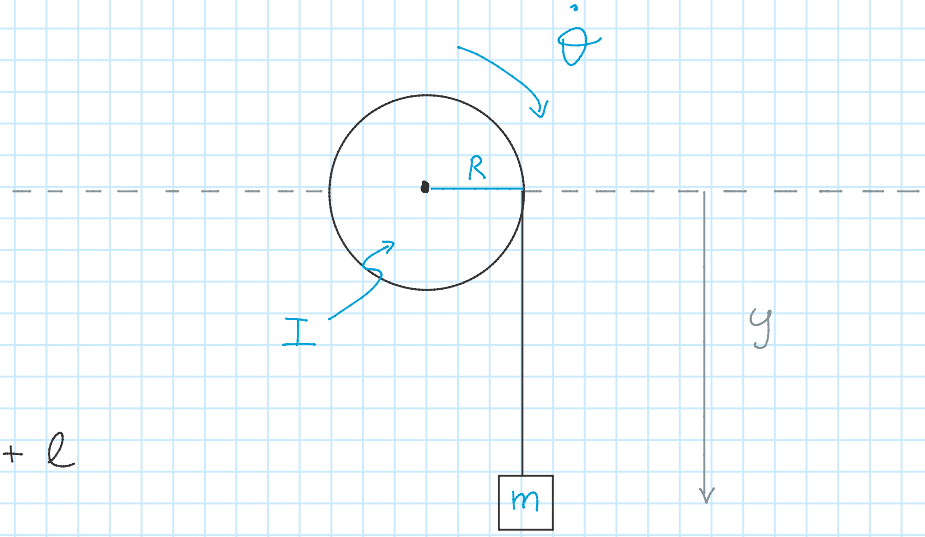
A mass m is suspended from a massless string, the other end of which is wrapped several times around a cylinder of radius R and moment of inertia I, which is free to rotate about a fixed horizontal axel. Using a suitable coordinate, set up the Lagrangian and the Lagrange equation of motion, and find the acceleration of the mass m.



$$y = R\theta + \ell$$

$$\mathcal{L} = \frac{1}{2} m (R\dot{\theta})^2 + \frac{1}{2} I \dot{\theta}^2 + mg(R\theta + \ell)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mR^2\dot{\theta} + I\dot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = mgR$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = (mR^2 + I)\ddot{\theta} - mgR = 0$$

$$\ddot{\theta} = \frac{mgR}{mR^2 + I} \longrightarrow \ddot{y} = \frac{mgR^2}{mR^2 + I}$$

With Newton's equations:

$I\ddot{\theta} = TR$ Newton's 2nd law for the pulley

$m\ddot{y} = mg - T$ Newton's 2nd law for the mass

$$T = \frac{I\ddot{\theta}}{R}$$

$$mR\ddot{\theta} = mg - \frac{I\ddot{\theta}}{R} \qquad \left(mR + \frac{I}{R}\right)\ddot{\theta} = mg$$

$$\boxed{\ddot{\theta} = \frac{mgR}{mR^2 + I}}$$

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