Bead on a straight wire

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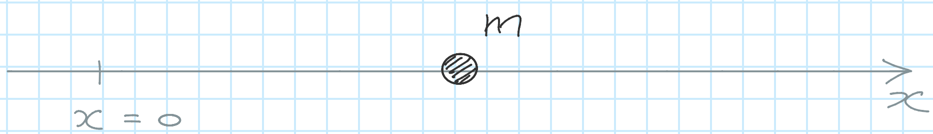
As a first simple example of the application of the Hamiltonian formalism, consider the case of a bead that slides without friction along a straight wire and it is furthermore subject to a conservative force described through a potential U(x). This force could be for example the restoring force provided by a spring.



The Lagrangian of the system is

$$\mathcal{L} = \frac{1}{2} m \dot{x}^2 - U(x)$$

$$\hookrightarrow \quad \frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} = 0 \longrightarrow m\ddot{x} = -\frac{dU}{dx}$$

One can then build the Hamiltonian and Hamilton's equations as follows

$$p = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}$$

$$\mathcal{H} = p\dot{x} - \mathcal{L} = \frac{p^2}{m} - \left[\frac{p^2}{2m} - U(x)\right] = \frac{p^2}{2m} + U(x)$$

$$\frac{\partial H}{\partial x} = -\dot{p} = \frac{dU}{dx} \qquad \dot{p} = -\frac{dU}{dx}$$

NEWTON'S 2nd LAW

$$\frac{\partial H}{\partial p} = \dot{x} = \frac{p}{m} \qquad p = m\dot{x}$$

definition of linear momentum