PHYS 3100 - Second exam

Problem 1

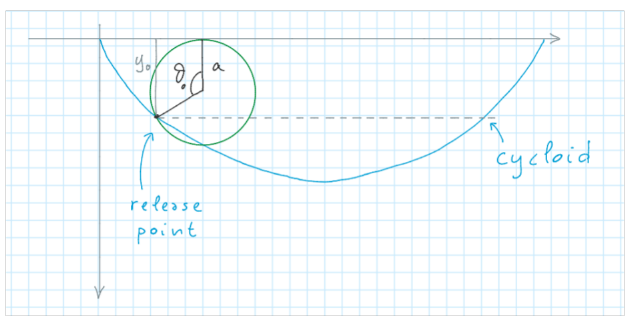
Consider a bowl shapes like a cycloid. Show that the oscillation period of a marble released at any point in the bowl is always the same, irrespectively from how far the release point is from the bottom of the bowl. In particular prove that the oscillation period is

$$T = 4\pi\sqrt{\frac{a}{g}} \, .$$

(20 points)

Remember that the cycloid is the set of points x and y that satisfy the equations

$$x = a\,(\theta - \sin\theta) \ , \qquad y = a\,(1 - \cos\theta) \ .$$



Problem 2

- Using the angle ϕ as generalized coordinate, write down the Lagrangian for a simple pendulum of length l suspended from the ceiling of an elevator that is accelerating upward with constant acceleration a. *(10 points)*

- Find the Lagrange equation of motion and show that it is the same as that for a normal, non accelerating pendulum, except that g has been replaced by $g + a$. In particular, the angular frequency of the small oscillations is $\sqrt{(g+a)/l}$. *(10 points)*

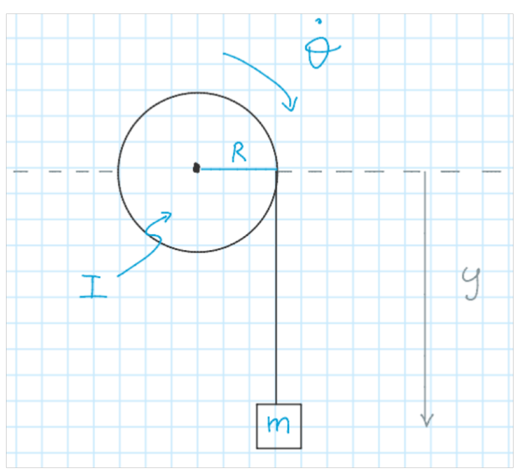
Problem 3

- A smooth wire is bent into the shape of an helix, with cylindrical polar coordinates $\rho = R$ and $z = \lambda\phi$, where R and λ are constants and the z axis is vertically up (and gravity vertically down). Using z as your generalized coordinate,write down the Lagrangian for a bead of mass m threaded on the wire. Find the Lagrange equation and hence the bead's vertical acceleration $\ddot{z}$. *(10 points)*
- In the limit $R \to 0$, what is $\ddot{z}$? Does this make sense? Why or why not? *(10 points)*

Problem 4

A mass m is suspended from a massless string, the other end of which is wrapped several times around a cylinder of moment of inertia I, which is free to rotate about a fixed horizontal axel.

- Using the angle of rotation of the cylinder θ as the generalized coordinate, find the Lagrangian of the system. *(10 points)*
- Write the Lagrange equation of motion and find the acceleration of the mass. *(10 points)*



Problem 5

Two masses m_1 and m_2 can move vertically and without friction along a pole. The two masses are joined by a spring of negligible mass and natural length L and force constant k. Initially m_2 is resting on the table while m_1 is held at a distance L above m_2. At the time $t = 0$, m_1 is projected upward with a velocity v_0.

a) Write the Lagrangian for the system formed by the masses by using the vertical location the center of mass (Y), and the distance between the two masses (y) as generalized coordinates. *(10 points)*

b) Solve the equations of motions for Y and y and then use those solutions to find the positions of the two masses at a generic time t. *(10 points)*