Snell's law

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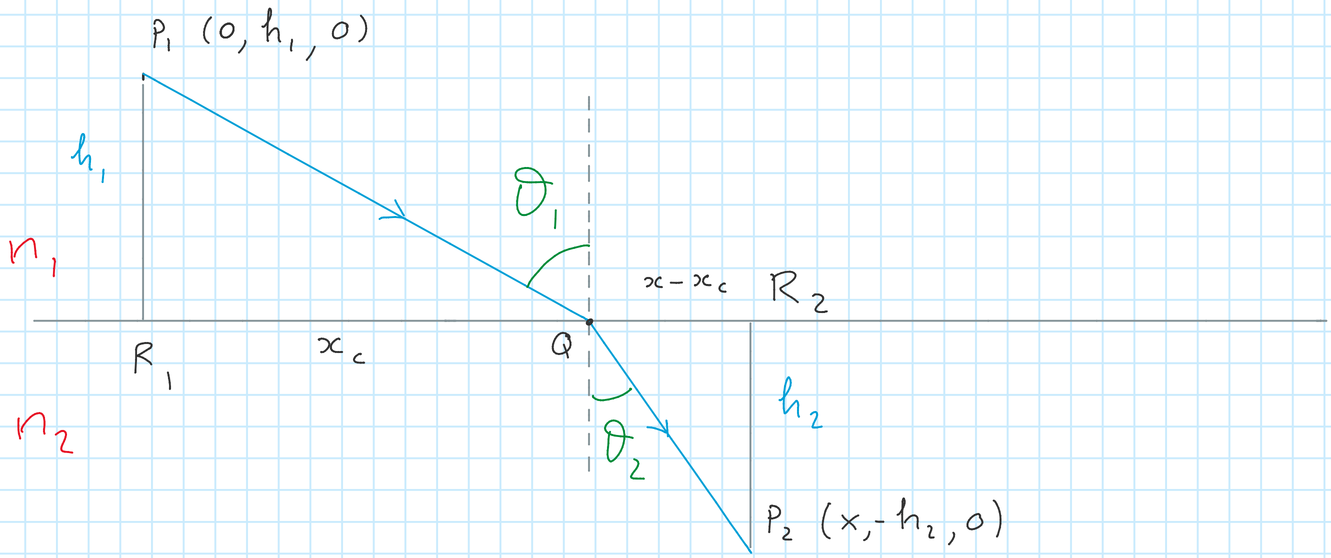
From Taylor problem 6.4

A ray of light travels from point P1 of coordinates (0,h1,0) in a medium of index of refraction n1 to a point P2 of coordinates (x,-h2,0) in a medium with an index of refraction n2. The interface between the two materials is on the x ,z plane. Show that Fermat's principle requires that the point where the ray leaves the first medium to enter the second medium, Q, is such that the ray of light satisfies Snell's law.



The point Q can have a priory coordinates (xc,0,z). We need to fix the coordinates xc and zc by requiring that the time of travel of the light beam between P1 and P2 is the shortest possible.

$$t_1 = \frac{\sqrt{x_c^2 + h_1^2 + z_c^2}}{v_1} = \frac{n_1}{c}\sqrt{x_c^2 + h_1^2 + z_c^2}$$

time of travel in the material of index n_1

$$t_2 = \frac{n_2}{c}\sqrt{(x - x_c)^2 + h_2^2 + z_c^2}$$

time of travel in the material of index n_2

$$t = t_1 + t_2 = \frac{n_1}{c}\sqrt{x_c^2 + h_1^2 + z_c^2} + \frac{n_2}{c}\sqrt{(x - x_c)^2 + h_2^2 + z_c^2}$$

$$\frac{\partial t}{\partial z_c} = 0 \longrightarrow z_c = 0$$

∂z_c

$$\frac{\partial t}{\partial x_c} = 0 \longrightarrow \frac{n_1}{c}\frac{x_c}{\sqrt{x_c^2+h_1^2}} - \frac{n_2}{c}\frac{x-x_c}{\sqrt{(x-x_c)^2+h_2^2}} = 0$$

$$\hookrightarrow \quad n_1 \frac{R_1 Q}{P_1 Q} - n_2 \frac{R_2 Q}{P_2 Q} = 0$$

$$\boxed{n_1 \sin\vartheta_1 - n_2 \sin\vartheta_2 = 0}$$

SNELL'S LAW