Lagrangian N unconstrained particles

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The extension of the formalism to the case of N unconstrained particles is straightforward. The position of each particle will be described by three generalized coordinates, so that the configuration space will have 3 N dimensions. A generalized velocity corresponds to each generalized coordinate. One can then write 3N Lagrange's equations that determine the motion of all of the particles in the system.

$$\boxed{\begin{aligned}&\mathcal{L}(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_n, \dot{\bar{r}}_1, \dot{\bar{r}}_2, \ldots, \dot{\bar{r}}_n) \\ &= \frac{1}{2} m_1 \dot{\bar{r}}_1^{\,2} + \frac{1}{2} m_2 \dot{\bar{r}}_2^{\,2} + \ldots + \frac{1}{2} m_N \dot{\bar{r}}_N^{\,2} - U(\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_N)\end{aligned}}$$

$$\{\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_N\} \equiv \{q_i\}$$

CONFIGURATION SPACE

$$\boxed{\frac{\partial \mathcal{L}}{\partial q_i} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0 \qquad i = 1, 2, \ldots, 3N}$$

3 N LAGRANGE EQS.