# Rocket with linear air drag

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Problem 3.14 Taylor

Suppose to have a rocket moving horizontally and subject to linear air drag. Assume that the rocket is burning fuel linearly. Find an equation for the velocity as a function of the rocket's residual mass. The rocket starts from rest.

Solution

$$m \dot{v} = -\dot{m} v_{ex} + F_{ext}$$

$$m \dot{v} = -\dot{m} v_{ex} - b v \qquad m \equiv m_0 - k t$$

$$k > 0$$

$$\frac{dv}{dt} = \frac{k v_{ex} - b v}{m_0 - k t}$$

$$\frac{dv}{k v_{ex} - b v} = \frac{dt}{m_0 - k t}$$

$$-\frac{1}{b} \ln\left(k v_{ex} - b v\right)\Big|_{v_0 = 0}^{v} = -\frac{1}{k} \ln\left(\frac{m_0 - k t}{m_0}\right)$$

$$\frac{1}{b} \ln\left(1 - \frac{b v}{k v_{ex}}\right) = \frac{1}{k} \ln\left(1 - \frac{k t}{m_0}\right)$$

$$\ln\left(1 - \frac{b v}{k v_{ex}}\right) = \frac{b}{k} \ln\left(\frac{m}{m_0}\right)$$

$$1 - \frac{bv}{kV_{ex}} = \left(\frac{m}{m_o}\right)^{\frac{b}{k}}$$

$$V = \frac{kV_{ex}}{b}\left[1 - \left(\frac{m}{m_o}\right)^{\frac{b}{k}}\right]$$