Equilibrium point

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Problem 5.13 in Taylor

The potential energy in a one dimensional central problem is

$$U(r) = U_0\left(\frac{r}{R} + \lambda^2 \frac{R}{r}\right) \qquad 0 < r < \infty$$

$$U_0 > 0 \qquad \lambda > 0$$

$$R > 0$$

Find the equilibrium position r_0. Let x be the distance from equilibrium and show that, for small x,

$$U = \frac{1}{2} k x^2 + \text{const}$$

Write k in terms of U_0 and R. Find the oscillation frequency ω

$\boxed{\text{Solution}}$

$$\frac{\partial U}{\partial x} = U_0\left(\frac{1}{R} - \lambda^2 \frac{R}{r^2}\right) = 0 \qquad r_0 = \lambda R$$

$$r = \lambda R + x$$

$$U = U_0\left(\lambda + \frac{x}{R} + \lambda \frac{1}{1 + \frac{x}{\lambda R}}\right)$$

$$= U_0\left[\lambda + \frac{x}{R} + \lambda\left(1 - \frac{x}{\lambda R} + \frac{x^2}{\lambda^2 R^2} + \dots\right)\right]$$

$$= U_0\left(2\lambda + \frac{x^2}{\lambda R^2} + \dots\right) = 2U_0\lambda + \frac{U_0}{\lambda R^2} x^2 \leftarrow \frac{1}{2} k x^2$$

$$k = \frac{2U_0}{\lambda R^2}$$

$$F = -\frac{\partial U}{\partial x} = -\frac{2U_0}{\lambda R^2}x$$

$$m\ddot{x} = -\frac{2U_0}{\lambda R^2}x \qquad \omega = \sqrt{\frac{2U_0}{mR^2\lambda}}$$

check dimensions

$$\left[\sqrt{\frac{2U_0}{\lambda m R^2}}\right] = \sqrt{\frac{kg\,\frac{m^2}{s^2}}{kg\,m^2}} = \frac{1}{s} \qquad \checkmark$$