Quadratic drag: Horizontal motion

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Let's consider the motion of an object moving along the positive x direction and subject to a quadratic drag force.

$$m\frac{dv}{dt} = -cv^2$$

This differential equation can be solved with the method of the separation of variables: All of the dependence on v should be confined to the l.h.s of the equation, while the dependence on t should be confined to the opposite side

$$m\frac{dv}{v^2} = -c\,dt$$

$$m\int_{v_0}^{v}\frac{dv'}{(v')^2} = -c\int_0^t dt'$$

$$m\left[-\frac{1}{v'}\right]_{v_0}^{v} = -ct \longrightarrow m\left(\frac{1}{v_0}-\frac{1}{v}\right) = -ct$$

One can now solve for v

$$\frac{m}{v_0} + ct = \frac{m}{v} \longrightarrow \frac{m+ctv_0}{v_0} = \frac{m}{v}$$

$$\boxed{v(t) = \frac{v_0}{1+\frac{cv_0}{m}t}}$$

VELOCITY AS A FUNCTION OF TIME

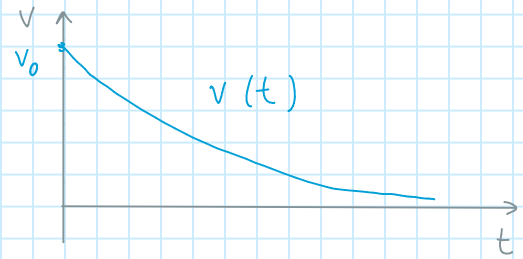
It is convenient to introduce the quantity tau that has the dimension of time

$$\tau \equiv \frac{m}{cv_0}$$

$$\left(\text{rem } [c] = \frac{N}{\frac{m^2}{s^2}} = \frac{Kg}{m},\ [\tau] = \frac{Kg}{\frac{Kg}{m}\frac{m}{s}} = s\right)$$

The equation for v can then be rewritten as

$$v(t) = \frac{v_0}{1 + \frac{t}{\tau}}$$

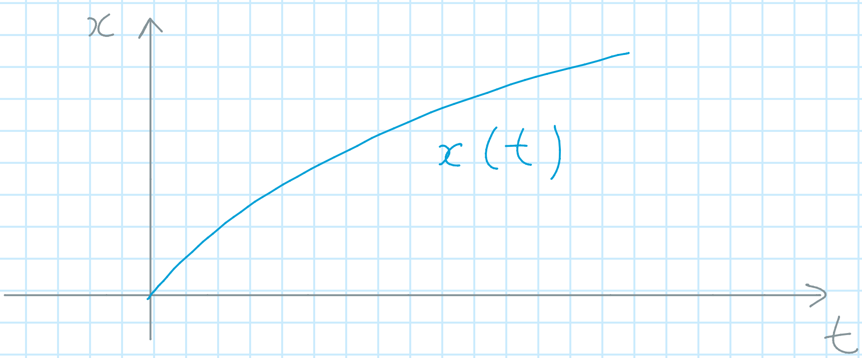


The next step consists in finding the position as a function of time

$$x(t) = x_0 + \int_0^t v(t')\,dt'$$

$$= x_0 + \int_0^t \frac{v_0}{1 + \frac{t'}{\tau}}\,dt'$$

$$= x_0 + v_0\tau \int_0^u \frac{du'}{1+u'} = \boxed{x_0 + v_0 \tau \ln\left(1 + \frac{t}{\tau}\right)}$$



Notice that in this case x grows indefinitely, even if only very slowly (logarithmically). In real life, when v becomes very small the linear drag becomes more important than the quadratic drag. We already showed that an object moving on a line and subject to a linear drag force will stop.

Why is linear drag more efficient at stopping and object than the quadratic drag? Because when v becomes small the quadratic drag becomes small much faster than the linear drag.