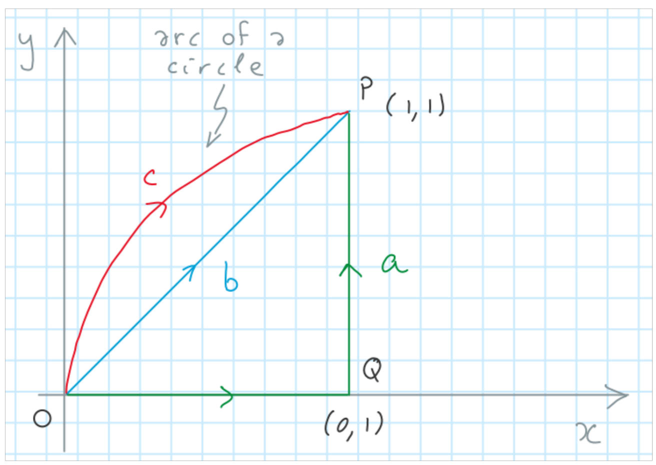
PHYS 3100 - First exam

Problem 1

Consider the two dimensional force

$$\mathbf{F} = y\hat{\mathbf{i}} + 2x\hat{\mathbf{j}}\,,$$

Calculate the work done by this force on a each one of the three paths going from the origin of the frame of reference to the point P of coordinates $(1, 1)$ and shown in the figure below.



a)

$$W_a = \int_a \mathbf{F} \cdot d\mathbf{r}\,, \qquad \textit{(10 points)}$$

b)

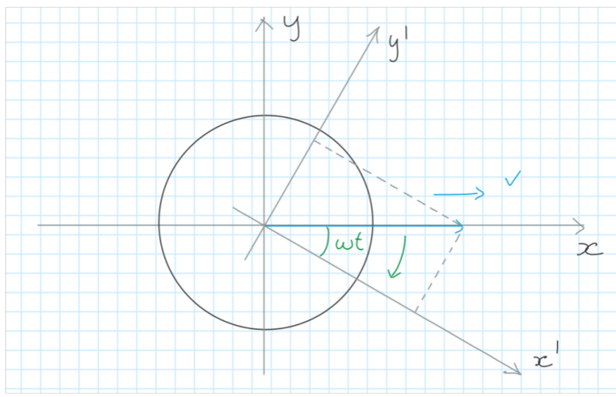
$$W_b = \int_b \mathbf{F} \cdot d\mathbf{r}\,, \qquad \textit{(10 points)}$$

c)

$$W_c = \int_c \mathbf{F} \cdot d\mathbf{r}\,, \qquad \textit{(20 points)}$$

Problem 2

Consider a puck moving with constant velocity v along the x axis on a frictionless surface. At the time $t = 0$ the puck is located at the origin of the frame of reference. Imagine that the surface below the puck is a turntable that rotates at constant angular velocity ω with respect to the direction of motion of the puck. The center of the rotation is the position of the puck at the time $t = 0$. When seen from above, the turntable rotates clockwise.



a) Write an equation for the coordinate x' and y' of the puck in the rotating frame. These coordinates are functions of v, ω and t. *(10 points)*

b) Plot the trajectory of the puck in the rotating frame. In the plot, set $v = 1$ m/s, $\omega = 1$ 1/s, and plot the trajectory for $t \in [-6\pi, 6\pi]$. *(10 points)*

Problem 3

The velocity of a rocket accelerating vertically from rest in a gravitational field g is given by

$$v(t) = -gt + v_{\text{ex}} \log\left(\frac{m_0}{m_0 - kt}\right) .$$

The initial mass of the rocket loaded with fuel is indicated by m_0, v_{ex} indicates the velocity of the exhaust with respect to the rocket, and k is a positive constant. Integrate the equation above to find an equation for the position y as a function of time. *(10 points)*

Problem 4

Consider an object that is coasting horizontally (positive x direction) subject to a drag force

$$f = -bv - cv^2 \,.$$

Write down Newton's second law for this object and solve for v by separating variables. (Indicate the initial velocity with v_0, check that at $t = 0$ your formula gives $v = v_0$.) What happens when $t \to \infty$? *(10 points)*

Problem 5

The potential energy of a particular mass m depends only on the distance from the center of the frame of reference and it is given by

$$U(r) = U_0 \left(\frac{r}{R} + \lambda^2 \frac{R}{r} \right) \,.$$

U_0, R and λ are all positive quantities.

a) Find the value of r at which the mass is in equilibrium, i.e. the value of r for which the force acting on the particle is 0. Let's indicate that particular value of r with r_0. Is the equilibrium point stable or unstable? *(10 points)*

b) Write $r = r_0 + x$. Show that for small x the potential is equal to

$$U(r) = \frac{1}{2} k x^2 + \text{const.} \,,$$

where k is a constant quantity. *(10 points)*

c) Calculate the angular frequency ω that characterizes the small oscillations around the equilibrium point. *(10 points)*