Newton's third law

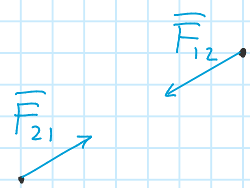
Monday, June 3, 2019 4:35 PM

Every force is applied by something (or somebody) on something (or somebody) else. The difference between what applies the force and the object on which the force is applied is crucial. Newton's third law links the forces applied by two objects on each other.

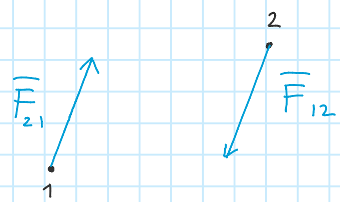
If object 1 exerts a force F_12 on object 2, then object 2 always exerts a reaction force F_21 on the object 1. The two forces are equal in magnitude but opposite in direction

$$\boxed{\bar{F}_{21} = -\bar{F}_{12}}$$

Case a)



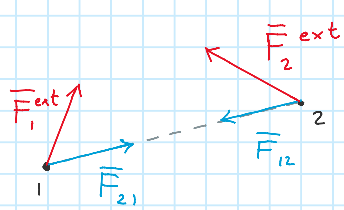
Case b)



Most forces fall under case a), but case b) is also sufficient to satisfy Newton's third law. Forces that fall under case a) are called central forces and they act along the line that joins the position of particle 1 and particle 2.

$\boxed{\text{Conservation of momentum: Two particle case}}$

Newton's third law of motion is equivalent to the statement that in a system of particles on which the net external force is zero the total momentum is conserved. Let's prove this first for a system of only two particles. On each one of the particles two forces are applied: An external force and a force applied by the other particle in the system.



From Newton's second law for the particles 1 and 2 one has

$$\dot{\bar{p}}_1 = \bar{F}_1^{ext} + \bar{F}_{21} \qquad \dot{\bar{p}}_2 = \bar{F}_2^{ext} + \bar{F}_{12}$$

One can then define the total momentum of the system

$$\bar{P} \equiv \bar{p}_1 + \bar{p}_2$$

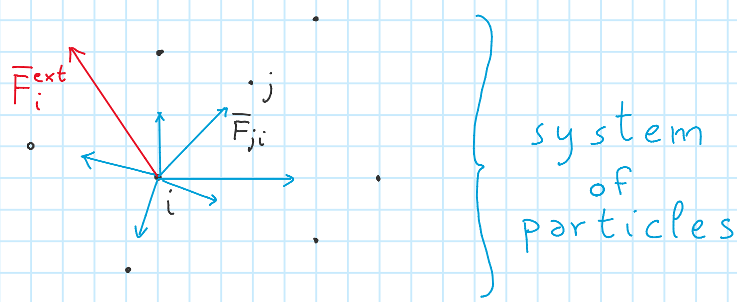
$$\dot{\bar{P}} = \dot{\bar{p}}_1 + \dot{\bar{p}}_2 = \underbrace{\bar{F}_1^{ext} + \bar{F}_2^{ext}}_{\bar{F}^{ext}} + \underbrace{\bar{F}_{21} + \bar{F}_{12}}_{\substack{=0 \\ \text{Newton's} \\ \text{third law}}}$$

$$\boxed{\dot{\bar{P}} = \bar{F}^{ext}}$$

if $\bar{F}^{ext} = 0 \longrightarrow \dot{\bar{P}} = 0 \longrightarrow$ momentum is conserved

Conservation of momentum: General case

The discussion of conservation of momentum can be easily generalized to the case of N particles. It is useful to go through the derivation in order to become familiar with the notation involving summations. Consider a particle out of a system including N particles. Let's indicate this particle with the index i



Newton's second law applied to the particle will be

$$\dot{\bar{p}}_i = \sum_{j \neq i} \bar{F}_{ji} + \bar{F}_i^{ext}$$

The total momentum of the system is

$$\bar{P} = \sum_{i=1}^{N} \bar{p}_i$$

$$\dot{\overline{P}} = \sum_{i=1}^{N} \dot{\overline{p}}_i = \sum_{i=1}^{N} \sum_{j \neq i} \overline{F}_{ij} + \sum_{i=1}^{N} \overline{F}_i^{ext}$$

One can now rewrite the double sum as follows

$$\sum_{i=1}^{N} \sum_{j \neq i}^{N} \overline{F}_{ij} = \sum_{i=1}^{N} \sum_{j > i}^{N} \underbrace{\left(\overline{F}_{ij} + \overline{F}_{ji} \right)}_{\substack{=0 \text{ Newton's} \\ \text{third law}}} = 0$$

Therefore

$$\dot{\overline{P}} = \sum_{i=1}^{N} \overline{F}_i^{ext} = \overline{F}^{ext}$$

if $\overline{F}^{ext} = 0 \longrightarrow \dot{\overline{P}} = 0 \longrightarrow$ momentum is conserved

In Newtonian mechanics conservation of momentum is a consequence of Newton's third law of motion. Is actually equivalent to Newton's third law. However conservation of momentum applies beyond classical mechanics; it remains valid in relativity and quantum mechanics.