Newton's first and second law of motion

Thursday, May 30, 2019 5:28 PM

It is convenient to start by considering Newton's laws of motion applied to an object which can be modeled as a point in space: A point particle. By definition this "object" has no width, depth, height and as such it cannot spin, but it can translate from one point to another. It only has translational motion.

Several physical situation can be satisfactorily modeled by assuming that a particular object is a point particle: For example an atom or a molecule in a gas can be considered as a point particle in certain problems, but also a planet orbiting the sun can be considered as a point particle if one is interested in determining the shape of the planet's orbit.

The first two of Newton's laws of motion state that

Newton's first law (a.k.a. Law of Inertia)

In absence of a net force acting on an object, the object moves with constant velocity. (remember velocity is a vector, it has a magnitude and a direction, it should not be confused with speed, which is only the magnitude of the velocity)

Alternative formulations

If the net force applied to a point particle is zero, a stationary particle remains stationary while a moving particle continues to move with the same speed in the same direction.

If the net force applied to a point particle is zero, the particle has zero acceleration.

Newton's second law of motion

The product of the mass of a particle and its acceleration is equal to the net force acting on the particle

$$m\bar{a} = \bar{F}$$

mass → m, acceleration → $\bar{a}$, net force → $\bar{F}$

Remember that

$$\bar{v} \equiv \frac{d\bar{r}}{dt} \equiv \dot{\bar{r}} \qquad \bar{a} \equiv \frac{d\bar{v}}{dt} = \frac{d^2\bar{r}}{dt^2} = \dot{\bar{v}} = \ddot{\bar{r}}$$

A point has a constant mass, therefore

$$\bar{F} = m\bar{a} = \frac{d}{dt}(m\bar{v}) = \frac{d\bar{p}}{dt} = \dot{\bar{p}}$$

This language makes it transparent that Newton's second law is a differential equation

$$\frac{d^2\bar{r}}{dt^2} = \frac{\bar{F}}{m}$$

By solving this second order differential equation one can find the position of the object as a function of time.

Lagrangian and Hamiltonian mechanics will give us new ways of getting directly to the relevant differential equations that we need to solve in order to study a physical system.

To begin with, let's solve one of the simplest cases of Newton's second law: An object can move in one direction and the net force applied on it is constant

$$\bar{r} \equiv x\,\hat{i} \qquad \bar{F} \equiv F_0\,\hat{i}$$

$$\frac{d^2\bar{r}}{dt^2} = \frac{\bar{F}}{m} \longrightarrow \frac{d^2x}{dt^2} = \frac{F_0}{m}$$

$$\dot{x}(t) = \int_0^t \ddot{x}(t')\,dt' = \int_0^t \frac{F_0}{m}\,dt' = v_0 + \frac{F_0}{m}t \qquad \text{velocity}$$

$$x(t) = \int_0^t \dot{x}(t')\,dt' = \int_0^t \left(v_0 + \frac{F_0}{m}t'\right)dt'$$

$$= x_0 + v_0 t + \frac{1}{2}\frac{F_0}{m}t^2 \qquad \text{position}$$

Not all differential equations in physics are that easy to solve...