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A new CPU cooler design based on an active cooling heatsink combined with heat pipes

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ABSTRACT

The performance of active CPU cooling heatsinks primarily depends on the forced air convection created by computer fans. Boosting the fan speed, however, results in noise, vibration problems, and increased power consumption. The active heatsink, therefore, should be optimized under the constraints of overall volume, cost, and noise level. In this paper, a new CPU cooler is proposed that provides a more efficient heat dissipation capacity from the CPU to a finned heatsink without adding more heat pipes at a low-noise level of a small fan under the confined space constraints of a computer chassis. Computational fluid dynamics simulations were used to search for a proper cooling design. The simulation results were validated with corresponding physical experiments. The proposed CPU cooler has been shown to provide a total thermal resistance of 0.11-0.19 °C/W at a noise level of 21.5-36.3 dBA.

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1. Introduction

CPU thermal problems have become an issue in computer industries and markets. One popular cooling solution uses aluminum extrusion heatsinks with aluminum or copper plate fins soldered to copper metal bases. Most research and development for obtaining solutions that would meet cooling requirements has concentrated on the use of a computer fan to maximize the heat transfer rate from the heatsink.

The rapid development of silicon technology, the demands for more powerful computing performance, the manipulation of extensive data, and increasing graphic processing capabilities have led to today's highly complex, super-fast CPUs running at beyond 3 GHz. These advances were made possible by the advent of nano-electronics with integrated circuits of nano-meter size that can be routinely fabricated with high precision and manufacturing yield [1]. On the other hand, the typical heatsink technology cannot cool down super-fast CPUs effectively under low-noise conditions. The performance of most of these heatsinks primarily depends on the forced air convection created by computer fans. The higher fan speed causes the noise level to reach high values of over 40 dBA, which has a significant adverse effect on the computer users.

The typically permissible operating temperature of a CPU is below 70 °C. The reliability of the chips then decreases by 10% for every 2 °C above the permissible operating temperature [2]. To maintain the temperature below the permissible limit, an air active CPU cooling system comprised of a fan, finned heatsink, and heat pipe combinations has been commercially used because it can dissipate heat under the reliable operation conditions of CPUs [3–5]. However, the active air cooling heatsinks have encountered some development problems such as those associated with manufacturing costs and limiting the system size to the confined space within a desktop PC chassis because CPU over-clocking and high-end personal computers require new CPU coolers that are able to eliminate 2–3 times as much heat as that eliminated by existing systems under low-noise conditions.

Typically, the total thermal resistance (R_t) is used to evaluate the thermal performance of CPU coolers. The CPU junction temperature and ambient temperature are kept constant and are represented as T_j and T_a , respectively. The heat dissipation (Q) of the CPU to the surrounding air can be expressed as

$$R_t = \frac{T_j - T_a}{Q} \tag{1}$$

Fig. 1 shows the general view of a thermal circuit for CPU coolers. R_c represents the contact resistance between the CPU and



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Fig. 1. Thermal circuit for CPU coolers based on a finned heatsink combined with heat pipes.

the heating plate of the cooler, R_b is the resistance associated with the heating plate, R_{hp} is the resistance associated with the heat pipe, and R_{fa} is the resistance associated with the finned heatsink and air convection. In addition, T_b is the bottom temperature of the heating plate and T_{fm} is the mean temperature of the finned heatsink. These resistances can be used to assess the performance of each component and improve the overall efficiency of the heat transfer process. Therefore, the total thermal resistance can also be defined as follows:

$$R_t = R_c + R_b + R_{hp} + R_{fa} \tag{2}$$

 R_c may be reduced by using thermal grease and decreasing the roughness of the bottom surface of the cooler [6]. R_{fa} is difficult to change because increasing the forced convection coefficient results in an increase in the acoustic level of the fan. Alternative means of decreasing R_{fa} requires a considerable increase in the surface area of the finned heatsinks, which results in a cost increase and conflicts with the recent trend toward compact computer chassis. Adding heat pipes enables performance improvement of the cooler. A heat pipe with an outer diameter of 6 mm can transfer between 20 and 50 W per pipe [7,8]. Therefore, to increase the heat dissipation considerably, the cooler must have either additional heat pipes or an increased fin area, both of which would produce a directly proportional increase in cost and system size. Both are undesirable, so the heat rejection step of CPU coolers primarily depends on the forced convection created by computer fans. To meet the demands of end users, designers have been forced to create even more innovative heatsink designs for CPU coolers that must satisfy in all aspects of enhanced cooling capacity, size limitation, and lower cost, all at a lower noise level.

In this study, we focus our attention on finding a new CPU cooler design that provides a more efficient heat performance at a lownoise level under the confined space constraints of a desktop computer chassis. The commercial computational fluid dynamics (CFD) simulation tool, SC/Tetra [13], was used to search for areas of improvement in one existing CPU cooler and to perform a simulation of the new CPU cooler, through which the airflow and thermal behavior regime of the newly devised finned heatsink in the cooler was confirmed. The new design provides an increase in cooling intensity to efficiently meet the lower thermal resistance and lower fan speed requirements. The results of the analysis were validated with corresponding physical experiments.

The rest of this paper is organized as follows: Section 2 presents details about a commercially available CPU cooler, CNPS-9700 from Zalman Tech Co., Ltd., consisting of an active cooling finned heatsink combined with heat pipes, and points out required improvements to enhance its thermal performance. Section 3 describes a new CPU cooler that employs a new finned-heatsink design; in this cooler, the disadvantages of the existing CPU cooler are rectified. Section 4 describes the experimental setups associated with the thermal performance and noise level tests. In addition, Section 4 compares the physical experiment results and the simulation results of the new CPU cooler and presents a comparative discussion of the experimental results of the existing and novel CPU coolers. Finally, Section 5 concludes this paper.

2. Existing CPU cooler

The CPU cooler employed for the comparative testing was the CNPS-9700 from Zalman. This cooler was selected because it is one of the best performing commercial coolers for CPUs [9]. Like most of the active coolers, it consists of a finned heatsink, a copper heating plate, three heat pipes, and a small axial fan, as shown in Fig. 2(a). One end of each heat pipe is embedded in the heating plate and acts as the evaporation section while the fined end acts as the condensation section. They are soldered to the heating plate and the fin array to reduce contact resistance. The CPU is mounted in contact with the heating plate. The heat load is transported via the heat pipes into the finned heatsink, which is then cooled using forced airflow.

As listed in Table 1, the dimensions of the CNPS-9700 are approximately $90 \text{ mm} \times 124 \text{ mm} \times 142 \text{ mm}$ (length \times wide \times height). It is embedded with heat pipes bent in a near figure "eight", the full circle at the top allowing for attachment of more fins. The fan blows air through all of the fins toward the rear. For maximum airflow coverage, the finned heatsink covers only the area beneath the blades to avoid a dead-zone, which is a tunnelshaped area behind the hub of the fan. This design can save weight and minimize the fin area. Additionally, along with the role of a fan shroud, the projecting parts of the fins placed on the front of the finned heatsink were designed to increase the surface area of the fins. To obtain fins with high thermal conductivity, a number of 125 thin (0.2 mm) copper fins is attached to each heat pipe. This design is able to deliver the cooling performance achieved by using up to six heat pipes in the CNPS-9700 by using just three heat pipes of diameter Ø6 mm.

The airflow pattern of the CNPS-9700 was simulated by the CFD code SC/Tetra 7.0. As depicted in Fig. 3(a), a 3-D computation domain was determined to avoid the blockage effects of axial-flow applied to the finned heatsink. The zero pressure condition was applied to the domain. The simulation was run under the steady-state condition. Using an arbitrary Lagrangian Eulerian method, the airflow simulation was carried out based on the active motion of the rotating fan blades, modeling the actual shape of the fan since the airflow pattern significantly affects finned heatsink performance [13]. Owing to the bent figure "eight" shape of the heat pipes, the symmetry analysis conditions cannot be applied to the CNPS-9700. As shown in Fig. 3(b), this simulation needs to include different sets of meshes. Most of the computational elements are hybrid unstructured meshes that include tetrahedral,



Fig. 2. (a) CNPS-9700 schematic diagram.

 Table 1

 Specifications of CPU cooling systems.

Parameters	CNPS-9700	New CPU cooler
Overall dimension	$90 \times 124 \times 142$	$94 \times 131 \times 152$
$(L \times W \times H)$, mm		
Overall weight, g	764	782
Material	Pure copper	
Heat pipe	3 ea of heat pipes	
	(Dia. Ø6 mm, 430 mm of length)	
Fin thickness, mm	0.2	
Fin pitch, mm	0.2	
Finned surface area, cm ²	5490	5402
Fan dimension (L \times W \times H), mm	$110\times110\times25$	$120\times120\times25$
Max. fan speed,	2800/39	2100/36
rpm/Noise level, dBA		

pyramid, and prime elements in order to enable not only the fan blades to move freely without having to generate and eliminate meshes but also to analyze the airflow pattern of the very narrow fin pitch (2 mm) of the whole finned heatsink. The meshes have 19,134,502 elements and 5,321,901 nodes. To reduce the computational period, the computation of this simulation was solved by using an IBM P5 595 at the Korea Institute of Science, Technology, and Information supercomputing center.

An upwind difference scheme and the Semi-Implicit Pressure Linked Equations (SIMPLE) algorithm were used to calculate the convection terms and the pressure-correction in the equation, respectively. The continuity and momentum equations of the airflow passing through the finned heatsink take the following forms [10–13]:

Continuity equation:

 $\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x_i} \rho u_i = 0$

Momentum equation:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho}{\partial x_j} u_j \rho u_i = -\frac{\partial}{\partial x_j} \left\{ \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \left(P + \frac{2}{3} \mu \frac{\partial u_k}{\partial x_{jk}} + \frac{\partial u_j}{\partial x_i} \right) \delta_{ij} \right\} + \rho g_i$$
(4)

The standard turbulence $k-\epsilon$ model was also used for the simulation [10–13]. The turbulent kinetic energy and its dissipation rate are computed using the following equation:

$$-\rho \overline{u_i u_j} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho k \delta_{ij}$$
(5)

Fig. 4 shows the airflow pattern of the CNPS-9700 simulated when the fan rotates at 2000 rpm under the steady-state condition. The high velocity flow passage inside the fin pitches along the "eight" shape of heat pipes. On the other hand, a low velocity flow pattern was found behind the hub of the axial fan. The dead-zone of the finned heatsink behind the hub implies that it is not necessary to increase the fin area to the rear of the hub. However, it is necessary to consider the flow regime related to the finned-heat pipe arrangement. Both heat pipe #1 and heat pipe #2 were affected by the airflow blown from the fan, but the velocity distribution over heat pipe #3 was poorer than that for the other two. The flow pattern around heat pipe #1 has a bad influence upon the heat transfer capacity of the other heat pipes because the airflow behind heat pipe #1 is not directed toward heat pipe #2. Thus, heat pipe #3 could not perform well. A staggered-finned-heat pipe arrangement is thereby required to boost the heat transfer capacity more efficiently. Nevertheless, it is difficult to apply the staggered arrangement to the CNPS-9700 at once. The current arrangement of the CNPS-9700 makes it easy to combine finned heatsink with heat pipes during the manufacturing process. The



(3)

Hybrid unstructured meshes

Fig. 3. Computational domain and meshes. (a) Computational domain. (b) Hybrid unstructured meshes.



Fig. 4. Velocity distribution and mesh diagram in accordance with the cross sections of CNPS-9700.

three heat pipes are simultaneously inserted into the finned heatsink. Those heat pipes are then bent to form the "eight" shape. This makes it possible to manufacture the CNPS-9700 without any complex processes. However, the staggered arrangement results in a complex manufacturing process. Due to the fact that the three heat pipes cannot be bent at a time to form the staggered arrangement, each and every one of fins must be inserted only subsequent to the fact that three heat pipes are bent.

3. Design and simulations of a new CPU cooler with novel design

A novel design for a CPU cooler was devised to improve thermal performance at low fan speeds and also be manufacturing friendly. With the tunnel shape advantage of the CNPS-9700, this novel design positions the fan in the center of the finned heatsinks. Airflow is then forced through the fins of one heatsink and exhausted from the fins of the other heatsink. The heat pipes are bent in an " Ω " like shape, providing almost a full circle to attach more fins. 268 thin (0.2 mm) copper fins are attached to each heat pipe. The dimensions of this novel cooler are approximately 94 mm × 131 mm × 152 mm (length × width × height).

Prior to prototyping, the new cooler was simulated to confirm it would provide the target heat dissipation—130 W of heat must be dissipated for a dual core Intel Pentium D 840 CPU running at 3.2 GHz. The model in SC/Tetra 7.0 was also used under the same simulation conditions to carry out computations for the new cooler. The meshes have 84,788,684 elements and 30,862,260 nodes. The thermal contact resistance between each part was ignored. The heat flux applied on the bottom of the heating plate was calculated using Q equals 130 W. Additional assumptions included that the thermal conductivity of the heat pipes was 400,000 W/m^2 [14] and that the fan was rotated at 900-2100 rpm when a voltage of 5-12 V was applied through the motherboard to the fan. As shown in Fig. 5, the airflow passage of the novel cooler develops both behind and in front of the fan and along the staggered-finned-heat pipe arrangement. The thermal behavior of the cooler is shown in the temperature contour plotted in Fig. 6. The heat transferred by each heat pipe is properly spread throughout the overall area of each fin. From the results of this simulation, the system thermal resistance



Fig. 5. Velocity distribution diagram in accordance with the cross sections of the new CPU cooler for a fan speed of 2000 rpm.

(R_s) of the new CPU cooler, which takes just R_c from R_t , was found to be around 0.076–0.17 °C/W and the heating plate temperature was below 55 °C when the heat source was 130 W and the ambient temperature was 30 °C (Fig. 7).

4. Experimental verification for the new design and discussion

The experimental apparatus was set up to evaluate the performance of the CPU coolers (Fig. 8). A dummy heater connected with a controllable electronic power supply unit was used instead of a real CPU. The heater consisted of a pure copper block spreader $(30 \times 30 \text{ mm})$ with an embedded carbon film resistor. In order to maximize the heat transfer into the heating plate, the carbon film resistor was soldered to the copper block spreader and encapsulated into the insulator with a 1.5 mm layer of glass fiber ($k = 0.046 \text{ W m}^{-1} \text{ K}^{-1}$) followed by 3.5 mm of bakelite ($k = 0.232 \text{ W m}^{-1} \text{ K}^{-1}$). The conduction heat losses through the back side of the dummy heater estimated from the temperature difference between the junction and the bakelite side exposed to air was less than 1%. Thermal grease was applied at the contact



Fig. 6. Temperature distribution diagram at a heat source of 130 W and a fan speed of 2000 rpm.



Fig. 7. Simulation results of new CPU cooler at a heat source of 130 W and an ambient temperature of 30 $^\circ\text{C}.$

between the dummy heater and the heating plate to reduce the contact thermal resistance. An axial fan with a controllable fan rpm unit dispersed heat in three ranges: silent mode (less than 23 dBA), normal mode (less than 32 dBA), and audible mode (more than 33 dBA). The local temperature at the dummy heater and the ambient temperature were physically measured by T-type thermocouples with an accuracy of ± 0.5 °C. The voltage and the current applied to the dummy heater were continuously monitored during the test to determine the applied power. All temperature data was automatically collected by a data acquisition system.

The noise test method is shown in Fig. 9. CPU coolers were set in the anechoic chamber where the background noise was 18 dBA. A microphone with an accuracy of ± 0.2 dBA was then set 1 m from the cooler with the adjustment angle inclined at 30°. All sound magnitude data were sent to a sound level meter that provided a digital readout. This test was carried out based on the standards of the Acoustical Society of America [15].

The system thermal resistance results for both coolers at a heat load of 130 W are given in Fig. 10. The physical experiments of the new cooler confirm that the simulation characteristics make qualitatively good results in observation. It is also seen that the new cooler had lower thermal resistance results than the CNPS-9700. As listed in Table 1, the two coolers have nearly the same finned



Fig. 8. Thermal performance evaluation apparatus for CPU coolers.



Fig. 9. Noise level evaluation apparatus for CPU coolers.

surface area, but there seems to be a significant performance difference between the two. As mentioned above, it is clear that the discrepancies in the experimental results of the two coolers take into account the heatsink design with the finned-heat pipe arrangement. As opposed to the one heatsink assembly of the CNPS-9700, the two heatsink assembly of the proposed design enhances the performance of all the heat pipes. The design ultimately leads to low thermal resistance at low fan speeds. The resultant characteristics of the proposed cooler at a total thermal resistance of 0.11-0.19 °C/W are superior to those of the CNPS-9700, as shown in Fig. 11; therefore, the new design with the staggered heat pipe arrangement produces better performance under the confined space constraints of a computer chassis without the addition of more heat pipes. This should keep down the cost of the CPU cooler. In fact, the manufacturing cost of the new cooler is very similar to that of the CNPS-9700 since the new design was created to be built as a variation of the commercially available manufacturing process of the CNPS-9700. Because the new design positions the fan in the center of the finned heatsinks, the noise level should be considered along with the thermal performance. Verifying the different thermal performance and acoustic results of



Fig. 10. System thermal resistance of CPU coolers in relation to fan speed.



Fig. 11. Total thermal resistance of CPU coolers in relation to fan speed.

the two coolers. Fig. 12 shows the dummy heater junction temperature with a range of increasing fan speeds. Each temperature and noise level result is plotted in Fig. 11 for both of the two coolers. The heater junction temperature of the CNPS-9700 was 57-49.7 °C at the noise range of 22-38.6 dBA. The new cooler, however, had a heater junction temperature of 55-45.2 °C at the noise level of 21.5-36.3 dBA. It is clear that the noise level of the new cooler is slightly lower than that of the CNPS-9700. Moreover, the new cooler being cooled at the medium speed (normal mode) of the fan showed a heater temperature slightly lower than that of the CNPS-9700 cooled at the maximum speed (audible mode). The noise level of the new cooler at the medium speed was less than 30 dBA from the noise test apparatus, about 23% lower than that of the CNPS-9700 at the maximum speed. In terms of the dummy heater junction temperature, the proposed cooler is found to be the best among the various CPU coolers reviewed in Fig. 13 (measured by the same physical experiment apparatus) because it has the most innovative configuration-heat load of 130 W, noise level of 35 dBA, and ambient temperature of 30 °C.



Fig. 12. Heater junction temperature and noise level of CPU coolers at a heat source of 130 W in relation to fan speed.



Fig. 13. Heater junction temperature of various CPU coolers at a heat source of 130 W, ambient temperature of 30 °C, and noise sound level of 35 dBA: (■) Intel CPU stock cooler (one 92 mm fan/no heat pipe/weight: 500 g); (▲) ZALMAN CNP510X (one 120 mm fan/6 ea of 6 mm U-shaped heat pipes/weight: 748 g); (■) ZALMAN CNP5-9700; (▲) NOCTUA NH-U12 (one 120 mm fan/5 ea of 6 mm U-shaped heat pipes/weight: 940 g); (●) NOCTUA NH-U14 (2 ea of 120 mm fan/6 ea of 6 mm U-shaped heat pipes/weight: 1240 g); (●) proposed cooler.

It is clear that the proposed novel design can provide CPUs with efficient transfer of a large amount of heat at a low-noise level because the cooling intensity of the new cooler is high at a low fan speed. Moreover, it is possible to reduce computer power consumption for the airflow-based forced convection by running the fan at its maximum speed.

5. Conclusion

This present paper proposes a new CPU cooler that makes it possible to obtain lower total thermal resistance values of 0.11–0.19 °C/W with lower noise levels of 21.5–36.3 dBA under confined space constraints without the addition of more heat pipes compared to current commercially available coolers.

To confirm the airflow regime of the existing CPU cooler, CFD simulations were performed. From the simulation, we could identify that a staggered-finned-heat pipe arrangement is required to boost the heat transfer capacity more efficiently. However, it is difficult to apply the form change quickly due to the complex manufacturing process. Thus, it was necessary to develop a new design for the CPU cooler that would improve the thermal performance at a low fan speed and would be easy to manufacture. On the basis of the tunnel shape of the existing CPU cooler, the new cooler was designed to position the fan in the center of two finned heatsinks combined with heat pipes bent in an " Ω " shape. Airflow is then forced through the fins of one heatsink and exhausted from the fins of the other heatsink. On the basis of the CFD simulation results, it was expected that the CPU cooler employing this new design could effectively cool CPUs because all the heat pipes combined with the two finned heatsinks performed well. The results of the physical experiments indicated that the total thermal resistance of the proposed cooler was between 0.11 and 0.19 °C/W while the noise level values were 21.5-36.3 dBA. The junction temperature values of the dummy heater, used instead of the real CPU, were 55-45.2 °C at a heat load of 130 W. All results indicate that the new cooler is superior to conventional coolers even though the finned surface area, the number of heat pipes, and the cooler dimension are very similar or equal to the conventional designs. The new design satisfies all requirements, namely, enhanced cooling capacity, confined size, low cost, and low-noise level.

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