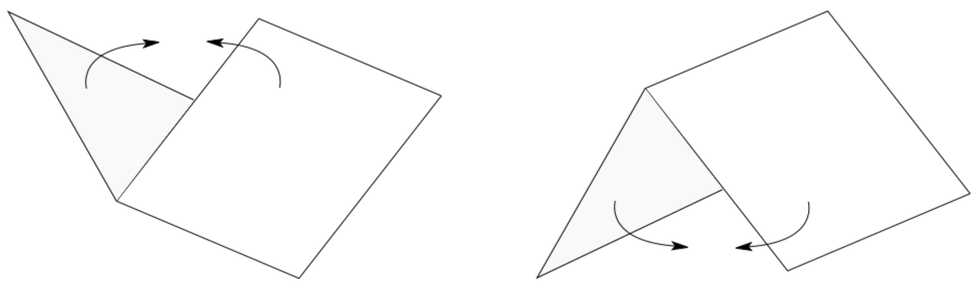
Dragon Fractal

To construct the fractal[1] known as the dragon, we will need a long strip of paper and a few definitions. A valley fold (V) on a piece of paper is a crease where the paper has been folded up towards itself (see Figure 1a) while a mountain fold (M) on a piece of paper is a crease where the paper has been folded down towards itself (see Figure 1b).



(a) A valley fold. (b) A mountain fold.

Figure 1: Valley and mountain folds.

To create the nth step dragon fractal, follow the steps below:

1. Begin by holding a long strip of paper with the ends in each hand.
2. Create a valley fold in the middle of the paper by bringing your right hand up and over towards your left hand.
3. Crease the paper in the middle.
4. Repeat steps 1, 2, and 3 with this new shorter strip of paper $n-1$ times.
5. Unfold the paper so that each fold is at $90°$ and place the paper on its edge (see Figure 2).

If we unfold the 1st step dragon fractal, we see that the paper contains only one fold, a valley fold, V. For the 2nd step dragon fractal, the unfolded paper contains 3 folds, VVM, in sequence from left to right. What sequence of folds appear on the paper (from left-to-right) on the 4th step dragon fractal? Can you predict the result of the 4th step based on the 3rd step? Let v_n count the number of valley folds in the nth step of the dragon fractal. Let m_n count the number of mountain folds in the nth step of the dragon fractal. Lastly, let $t_n = v_n + m_n$ (the total number of folds on the nth step of the dragon fractal).

1. Find a recurrence relation and initial conditions for v_n, m_n, and t_n when $n \geq 1$.

[1] `https://en.wikipedia.org/wiki/Fractal`

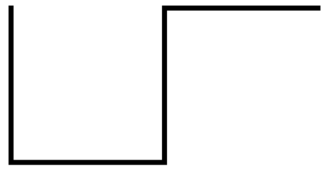


Figure 2: The 2nd step dragon fractal.

2. Solve each of the recurrence relation with initial conditions from part 1.
3. Sketch the 4th step dragon fractal.

For an online implementation of the dragon fractal, use the following link:

`https://trinket.io/python/01eb402472`.