MAT2540 - Partitions

A **partition** of a positive integer n is a way to write n as a sum of positive integers where the order of the terms in the sum does not matter. For example, $6 = 3 + 2 + 1$ is a partition of 6. Let p_m equal the number of different partitions of m, and let $p_{m,n}$ be the number of different ways to express m as the sum of positive integers not exceeding n.

1. Explain why $p_m = p_{m,m}$.

2. It is known that

$$p_{m,n} = \begin{cases} 1 & \text{if } m = 1 \\ 1 & \text{if } n = 1 \\ p_{m,m} & \text{if } m < n \\ 1 + p_{m,m-1} & \text{if } m = n > 1 \\ p_{m,n-1} + p_{m-n,n} & \text{if } m > n > 1. \end{cases}$$

   Write a program that computes p_m using the above formula for $p_{m,m}$. Hint: Write two functions, one that computes p_m as $p_{m,m}$ and a function that computes $p_{m,m}$ recursively. (see the block of code below)

```
def partition(m):
    return p(m,m)

def p(m,n):
    #Write recursive definition of p(m,n) here.
```

3. Compute $p_1, p_2, \ldots, p_{30}$. Use the following link to check your work: `https://oeis.org/A000041`.