Chapters 3.1: Slope of a Line and Finding the Equation of a Line

MAT 1275CO
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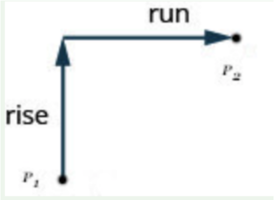
Slope of a Line

When we graph linear equations, we may notice that some lines tilt up and some lines tilt down as they go from left to right. Some lines are very steep and some lines are flatter.

In mathematics, the measure of the steepness of a line is called the slope of the line.

Find the Slope of a Line

Given two points P_1 and P_2 on a line, the **rise** is the vertical "distance" and the **run** is the horizontal "distance" traveled and moving from P_1 to P_2, as shown below.



The "distance" is positive when we are moving up or to the right, and negative when we are moving down or to the left.

The **slope of a line** is $m = \frac{\text{rise}}{\text{run}}$.

Find the Slope of a Line

The slope of the line between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

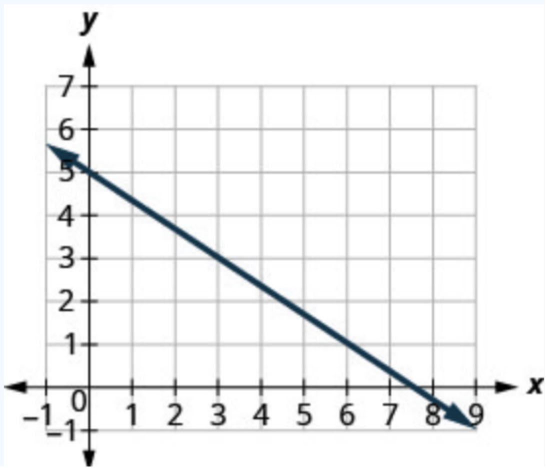
$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

- If we are given a graph of a line and ask to find the slope of the line, we can choose any two points on the line and use this formula to calculate the slope.
- If we are given two points on the line and asked to find the slope, we simply use the formula.

Example: Find the slope of the line through the points $(-2, 6)$ and $(-3, -4)$.

Find the Slope of a Line

Find the slope of the line below using both formulas.



Find the Slope of a Line

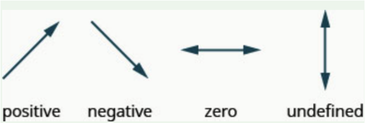
- The slope of a **horizontal line**, $y = b$, is 0.
- The slope of a **vertical line**, $x = a$, is undefined.

Example: Find the slope of each line:

- $x = 8$
- $y = -5$

Sign of the Slope

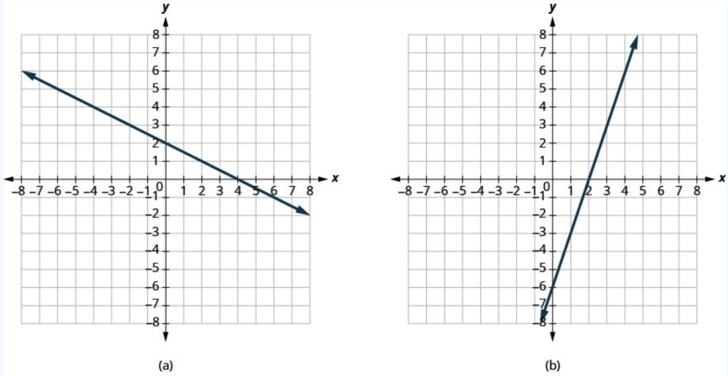
How does the sign of the slope determine the behavior line?



- If the slope is positive, then as the x-coordinates of the ordered pair solutions are increasing, the y-coordinates are increasing.
- If the slope is negative, then as the x-coordinates of the ordered pair solutions are increasing, the y-coordinates are decreasing.
- If the slope is zero, then as the x-coordinates of the ordered pair solutions are increasing, the y-coordinates remain the same.
- If the slope is undefined, then as the y-coordinates of the ordered pair solutions are increasing, the x-coordinates remain the same.

Sign of the Slope

What is the sign of the slope of the following lines? Find the slope.



Graph the line passing through $(0, 5)$ with slope $-\frac{3}{4}$ (by making use of the $m = \frac{\text{rise}}{\text{run}}$ formula).

Slope-Intercept Form

The **slope-intercept form** of an equation of a line with slope m and y-intercept $(0, b)$, is $y = mx + b$.

Example: Identify the slope and y-intercept from the equation of the line.

- $y = \frac{2}{5}x - 1$
- $x + 4y = 8$
- $3x + 2y - 12 = 0$

Example: Graph the line $y = -x - 3$ using its slope and y-intercept.

Graphing Lines

To recap what we have learned so far:

<table>
<tr><th colspan="4">Methods to Graph Lines</th></tr>
<tr><th>Point Plotting</th><th>Slope–Intercept</th><th>Intercepts</th><th>Recognize Vertical and Horizontal Lines</th></tr>
<tr>
<td><table><tr><th>x</th><th>y</th></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td>
<td>$y = mx + b$</td>
<td><table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td></td></tr><tr><td></td><td>0</td></tr><tr><td></td><td></td></tr></table></td>
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<tr>
<td>Find three points. Plot the points, make sure they line up, then draw the line.</td>
<td>Find the slope and y-intercept. Start at the y-intercept, then count the slope to get a second point.</td>
<td>Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.</td>
<td>The equation has only one variable.
$x = a$ vertical
$y = b$ horizontal</td>
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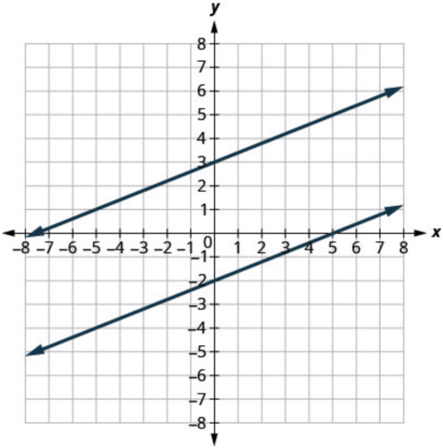
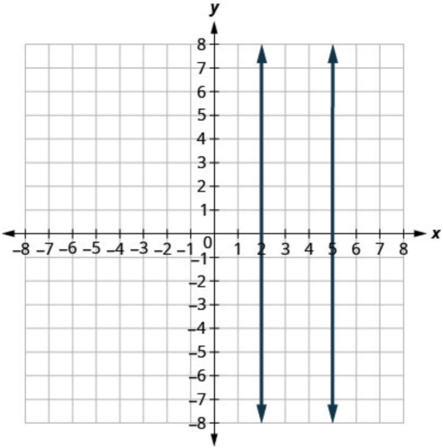
Parallel and Perpendicular Lines

Parallel lines are lines that have the same steepness and never intersect (have different y-intercepts).



Parallel and Perpendicular Lines

What about vertical lines? The slope of a vertical line is undefined but we see that vertical lines that have different x-intercepts are parallel, like the lines shown in this graph.



Parallel and Perpendicular Lines

Parallel lines are lines in the same plane that do not intersect.

- Parallel lines have the same slope and different y-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
- Parallel vertical lines have different x-intercepts.

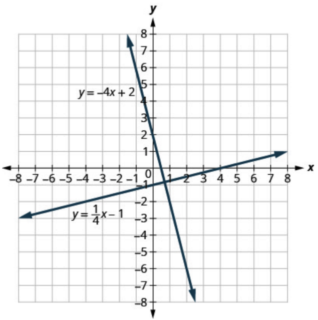
Example: Use slopes and y-intercepts to determine if the lines are parallel:

- $3x - 2y = 6$ and $y = \frac{3}{2}x + 1$
- $y = 2x - 3$ and $-6x + 3y = -9$
- $x = -2$ and $x = 5$
- $y = 3$ and $y = 7$

Parallel and Perpendicular Lines

Perpendicular lines are lines in the same plane that intersect at a right angle.

- If m_1 and m_2 are the slopes of two perpendicular lines, then their slopes are negative reciprocals of each other, $m_1 = -\frac{1}{m_2}$. In other words, the product of their slopes is -1, that is, $m_1 m_2 = -1$.
- A vertical line and a horizontal line are always perpendicular to each other.



Parallel and Perpendicular Lines

Example: Use slopes to determine if the lines are perpendicular:

- $y = -5x - 4$ and $x - 5y = 5$
- $7x + 2y = 3$ and $2x + 7y = 5$
- $x = -2$ and $y = 5$

On Your Own : Are the lines parallel, perpendicular, or neither?

- the line passing through the point (-1,4) with slope 1
- $6x + 6y - 6 = 0$

Find an Equation of a Line: Slope and y-intercept

If we have an equation of a line which is in slope-intercept form, $y = mx + b$, we can easily determine the line's slope and y-intercept.

Example:

- Find the slope-intercept form of an equation of a line with slope -8 and y-intercept $(0, -4)$.
- Find the slope-intercept form of an equation of a line with slope $\frac{2}{7}$ and y-intercept $(0, 3)$.

Point-Slope Form

The **point-slope form** of an equation of a line with slope m and containing the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

Using this form of an equation of a line allows us to find equations for lines with the slope and any point.

To find an equation of a line given the slope and a point

1. Identify the slope and the point.
2. Substitute the values into the point-slope form.
3. Write the equation in slope-intercept form or in the form $x = a$.

Point-Slope Form

Example:

- Find an equation of a horizontal line that contains the point $(-2, -6)$. Write the equation in slope-intercept form.
- Find an equation of a line with slope $-\frac{5}{8}$ and passing through the point $(16, 4)$. Write the equation in slope-intercept form.

Finding an Equation of the Line Given Two Points

To find an equation of a line given two points

1. Find the slope using the given points: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2. Choose one point.
3. Substitute the values into the point-slope form: $y - y_1 = m(x - x_1)$.
4. Write the equation in slope-intercept form.

Example:

- Find the equation of the line that contains the points $(-3, 5)$ and $(-3, 8)$.
- Find the equation of the line that contains the points $(2, 4)$ and $(4, 8)$.
- **On Your Own**: Find the equation of the line that contains the points $(-2, 10)$ and $(2, 0)$.

Finding an Equation of the Line

Now we can use everything we know to answer the following problems.

Example: For the following problems, write the equation in slope-intercept form, or in the form $x = a$.

- Find the equation of the line that is perpendicular to the line $x + 4y = 16$ and passes through the point $(1, -2)$.
- Find the equation of the line that contains the point $(6, 4)$ and is parallel to the line $y = \frac{1}{2}x - 3$.
- **On Your Own**: Find the equation of the line that is perpendicular to the line $x = 7$ and passes through the point $(7, -5)$.