Chapters 3.1: Linear Equations with Two Variables

MAT 1275CO
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Linear Expressions and Linear Equations with One Variable

Recall:

1. An expression that can be written as

$$Ax + B$$

with A and B real numbers, $A \neq 0$, is called a **linear expression (with one variable)**, or more specifically, a **linear expression in x**.

2. An equation that can be written as $Ax + B = 0$ with A and B real numbers, $A \neq 0$, is called a **linear equation (with one variable)**, or more specifically, a **linear equation in x**.

3. A **solution** to a linear equation with one variable, say x, is a number, say a, that when substituted in for that variable yields a true statement. In this case we say that $x = a$ is a solution.

Linear Expressions and Linear Equations with One Variable

Recall:

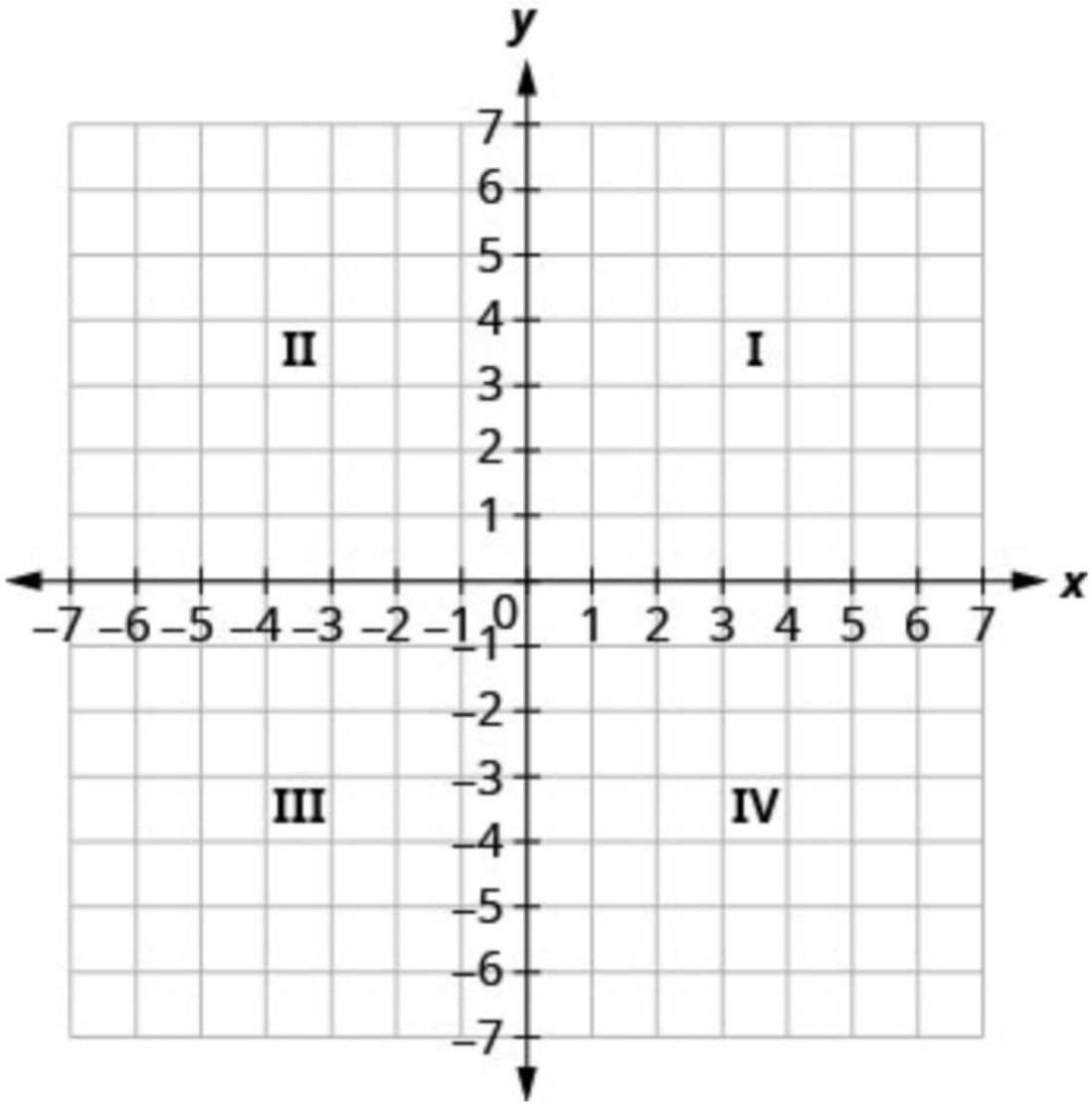
- Is $x^2 - 3x + 14 = 2 + x^2 - 5x$ a linear equation?
- Solve for x: $8 - 3x = 20$
- Evaluate $3x - 2y$ when $x = 4$ and $y = -3$.

Plotting Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system, or a rectangular coordinate system, is used in algebra to represent ordered pairs of numbers, and ultimately, to show a relationship between two variables. The rectangular coordinate system is also called the xy-plane or the "coordinate plane."

The rectangular coordinate system is formed by two intersecting number lines, one horizontal and one vertical. The horizontal number line is called the x -axis. The vertical number line is called the y-axis (in most contexts). These axes divide a plane into four regions, called quadrants. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise.

Quadrants



Plotting Points on a Rectangular Coordinate System

An **ordered pair**, (x, y), gives the coordinates of a point in a rectangular coordinate system. The first number is the x-coordinate. The second number is the y-coordinate.

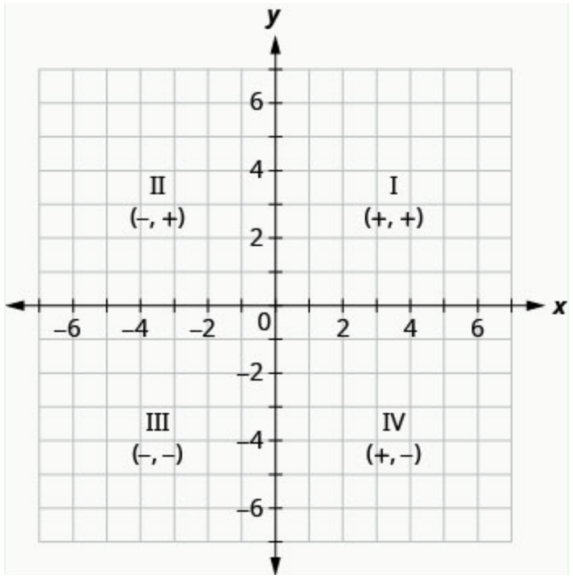
The point $(0, 0)$ is called the origin. It is the point where the x-axis and y-axis intersect.

We use the coordinates to locate a point on the xy-plane. Let's plot the point $(1, 3)$.

Plotting Points on a Rectangular Coordinate System

The signs of the x-coordinate and y-coordinate affect the location of the points.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$



Plotting Points on a Rectangular Coordinate System

Points on the x- or y-axis:

1. Points with a y-coordinate equal to 0 are on the x-axis, and have the form $(p, 0)$, where p is some real number.
2. Points with a x-coordinate equal to 0 are on the y-axis, and have the form $(0, p)$, where p is some real number.

Example: Plot each point in the rectangular coordinate system and identify the quadrant in which the point is located.

- A: $(-5, 4)$
- B: $(-3, -4)$
- C: $(2, 3)$
- D: $(0, -1)$
- E: $(0, 2)$

Linear Expressions and Linear Equations with Two Variables

1. An expression that can be written as

$$Ax + By$$

with A and B real numbers, not both zero, is called a **linear expression (with two variables)**, or more specifically, a **linear expression in x and y**.

2. An equation that can be written as

$$Ax + By = C$$

with A and B real numbers, not both zero, is called a **linear equation (with two variables)**, or more specifically, a **linear equation in x and y**.

Linear Expressions and Linear Equations with Two Variables

A linear equation with two variables, x and y, is in **standard form** when it is written as $Ax + By = C$.

An ordered pair (p, q) is a solution of the **linear equation** $Ax + By = C$, if the equation is a true statement when the x- and y-coordinates of the ordered pair, p and q, respectively, are substituted into the equation. We can also say in this case that $(x, y) = (p, q)$, or $x = p$ and $y = q$ is a solution.

Example: Is $(2, 4)$ a solution of the equation $3x + 5y = 36$? Is $(-1, 8)$?

Linear Expressions and Linear Equations with Two Variables

The **graph of the linear equation** $Ax + By = C$ is the collection of all solutions (x, y).

We can represent the graph on the coordinate plane. The **representation** is a straight line so that

- every solution of the equation is a point on this line, and
- every point on the line is a solution of the equation.

As universally accepted, a representation is also called a **graph of the linear equation**.

Linear Expressions and Linear Equations with Two Variables

Let's look at a graph of $y = 2x - 4$.

Example: For each ordered pair,

$$A : (0, -4) \qquad B : (4, 4) \qquad C : (2, -4) \qquad D : (-1. - 6)$$

decide:

1. is the ordered pair a solution to the equation?
2. is the point on the line?

Graph a Linear Equation by Plotting Points

There are several methods that can be used to graph a linear equation. The first method we will use is called plotting points. We find three points whose coordinates are solutions to the equation and then plot them in a rectangular coordinate system. By connecting these points in a line, we have the graph of the linear equation. While two points are enough to determine a line, using three points helps us detect errors.

Graph a linear equation by plotting points

1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.
3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

Graph a Linear Equation by Plotting Points

Example: Graph the following equations.

- $y = -2x + 4$
- $y = \frac{1}{2}x - 2$
- $y = 3x$

What is the difference between $y = 3x$ and $y = 3$? Graph $y = 3$ in the same rectangular coordinate system as $y = 3x$.

Graph Vertical and Horizontal Lines

Some linear equations have only one variable. They may have just x and no y, or just y without an x.

1. A **vertical line** is the graph of an equation (with two variables x and y) of the form $x = a$. The line passes through the x-axis at $(a, 0)$.
2. A **horizontal line** is the graph of an equation (with two variables x and y) of the form $y = b$. The line passes through the y-axis at $(0, b)$.

Example: Graph the following equations.

- $y = -4$
- $x = 1$

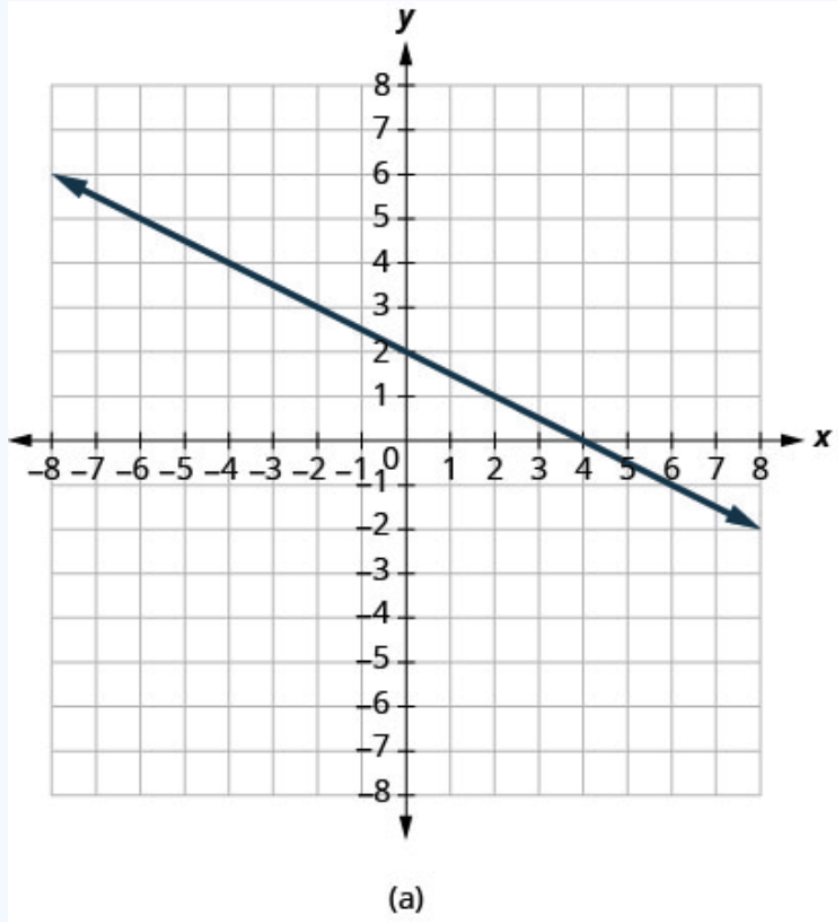
Find x- and y-intercepts

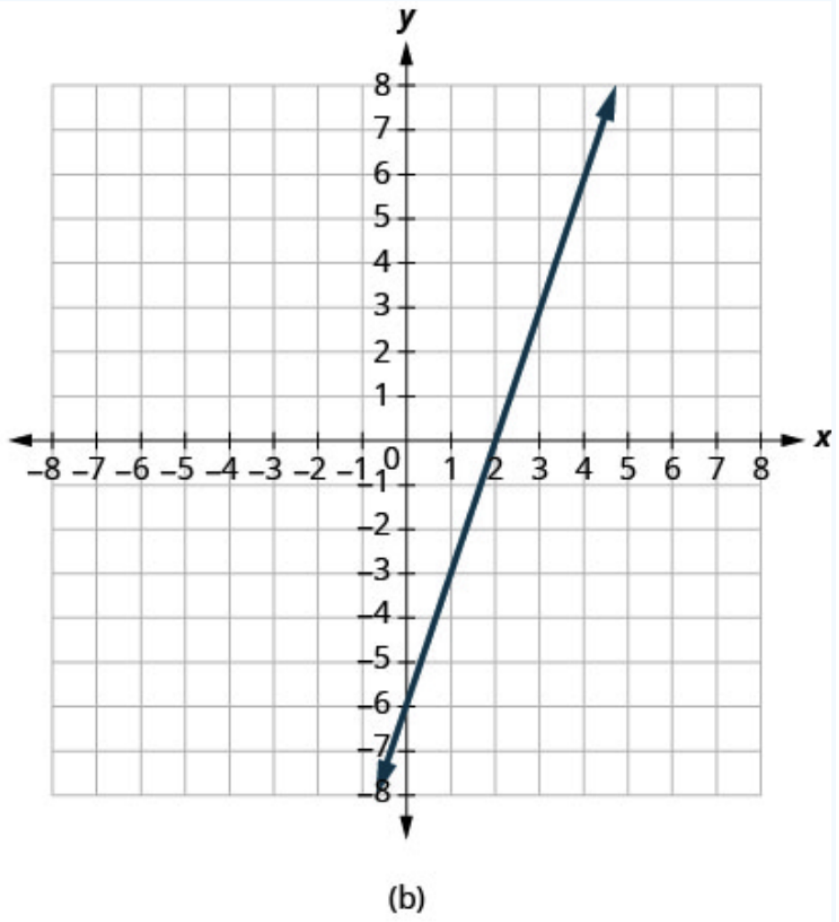
The points where a graph crosses the x- and y-axis are called the **intercepts of the graph**.

1. The x-intercept of a line is the point $(a, 0)$ where the line crosses the x-axis.
2. The y-intercept of a line is the point $(0, b)$ where the line crosses the y-axis.

Find x- and y-intercepts

Find x- and y-intercepts on each graph shown.





Find x- and y-intercepts

Intercepts from the equation of a line

To find:

- the x-intercept of the line, let $y = 0$ and solve for x.
- the y-intercept of the line, let $x = 0$ and solve for y.

When the line passes through the origin, the x-intercept and the y-intercept are the same point.

Example: Find the intercepts of the following graphs.

- $2x + y = 8$
- $x = 1$

Graph a Linear Equation by Plotting Points

Often, using the x- and y-intercepts as two of the three points for plotting makes graphing linear equations much simpler.

Example: Graph the following equations using the intercepts.

- $y = -x$
- $x - 2y = 4$
- **On Your Own**: $\frac{1}{3}x + y = 2$