Chapters 2.2.2 - 2.2.3: Solving Quadratic Equations Using the Square-Root Property and by Completing the Square

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Square Root Property

Square Root Property: If $x^2 = k$, then

$$x = \sqrt{k} \quad \text{or} \quad x = -\sqrt{k}.$$

These two solutions are often written

$$x = \pm\sqrt{k}.$$

Notice that the Square Root Property gives two solutions to an equation of the form $x^2 = k$, the principal square root of k and its opposite.

Examples: Solve.

- $x^2 = 9$
- $x^2 - 48 = 0$
- **On Your Own**: $3z^2 - 108 = 0$

Sometimes, we will need to use imaginary numbers in our answers. We should always check our work.

Solve a Quadratic Equation Using the Square Root Property

1. Isolate the quadratic term and make its coefficient one.
2. Use Square Root Property.
3. Simplify the radical.
4. Check the solutions in order to detect errors.

Examples: Solve.

- $c^2 + 12 = 0$
- **On Your Own**: $q^2 = -24$

Square Root Property

The solutions to some equations may have fractions inside the radicals. When this happens, we must rationalize the denominator.

Examples: Solve using the Square Root Property.

- $2x^2 - 8 = 41$
- **On Your Own**: $5q^2 - 36 = 0$

Quadratic Equations of the Form $a(x-h)^2 = k$

We can use the Square Root Property to solve an equation of the form $a(x-h)^2 = k$ as well. Notice that the quadratic term, x, in the original form $ax^2 = k$ is replaced with $(x-h)$.

$$ax^2 = k \qquad a(x-h)^2 = k$$

The first step, like before, is to isolate the term that has the variable squared. In this case, a binomial is being squared. Once the binomial is isolated, by dividing each side by the coefficient of a, then the Square Root Property can be used on $(x-h)^2$.

Quadratic Equations of the Form $a(x-h)^2 = k$

Examples: Solve using the Square Root Property.

- $4(y-7)^2 = 48$
- $\left(x - \frac{1}{2}\right)^2 = \frac{5}{4}$
- **On Your Own**: $(2t-8)^2 + 3 = -7$

Solving Quadratic Equations by Completing the Square

In previous slides, we were able to use the Square Root Property to solve the equation $(y-7)^2 = 12$ because the left side was a perfect square. What happens if the variable is not part of a perfect square? Can we use algebra to make a perfect square?

Let's expand $(y-7)^2$ and $(x+9)^2$ to see if we notice a pattern.

Solving Quadratic Equations by Completing the Square

Binomial Squares Pattern: If a and b are real numbers,

$$(a+b)^2 = a^2 + 2ab + b^2,$$

and

$$(a-b)^2 = a^2 - 2ab + b^2.$$

We can use this pattern to “make” a perfect square.

Solving Quadratic Equations by Completing the Square

Complete a Square of $x^2 + bx$

1. Identify b, the coefficient of x.
2. Find $\left(\frac{1}{2}b\right)^2$, the number to complete the square.
3. Add the $\left(\frac{1}{2}b\right)^2$ to the $x^2 + bx$.
4. Factor the perfect square trinomial, writing it as a binomial squared.

Examples: Complete the square. Then write the result as a binomial squared.

- $x^2 - 26x$
- $y^2 - 9y$
- **On Your Own**: $n^2 + \frac{1}{2}n$

Solving Quadratic Equations by Completing the Square

In solving equations, we must always do the same thing to both sides of the equation. This is true when we solve a quadratic equation by completing the square too.

Solve a Quadratic Equation of the Form $x^2 + bx + c = 0$ by Completing the Square

1. Isolate the variable terms on one side and the constant terms on the other.
2. Find $\left(\frac{1}{2}b\right)^2$, the number needed to complete the square. Add it to both sides of the equation.
3. Factor the perfect square trinomial, writing it as a binomial squared on the left and simplify by adding the terms on the right.
4. Use the Square Root Property.
5. Simplify the radical and then solve the two resulting equations.
6. Check the solutions.

Solving Quadratic Equations by Completing the Square

Examples: Solve by completing the square

- $x^2 + 8x = 48$
- $x^2 + 10x + 4 = 15$
- **On Your Own**: $(x + 3)(x + 5) = 9$