Chapters 2.3 - 2.4: Polynomial Equations and Rational Equations

MAT 1275CO
Dr. Davie

Spring 2024

Review

Recall:

- Solve. $7(2x-3)(x-4)(3x+1)=0$
- Use long division to divide $(x^3-6x^2+11x-6)$ by $(x-1)$.

A number c is a **root** of a polynomial of a single variable if when we substitute c for that variable the result is equal to 0.

Example:

- What are the roots of the polynomials $7(2x-3)(x-4)(3x+1)$ and $(x^3-6x^2+11x-6)$?

Polynomial Equations

If c is a root of a polynomial, then $(x - c)$ is a factor of that polynomial, i.e., the polynomial is equivalent to $(x - c)$(some polynomial).

Examples:

- If $x = -1$ is a solution of the equation $x^3 + 2x^2 - 11x - 12 = 0$, what is one factor of the polynomial $x^3 + 2x^2 - 11x - 12$? Factor the polynomial completely and solve the equation.
- What are the factors of $(x^2 + 2x - 5)$?
- What are the factors of $(x^3 - x^2 - 30x)$?
- Find all solutions of the polynomial $(x^2 + 2x - 5)(x^3 - x^2 - 30x) = 0$.

Polynomial Equations

Examples:

- Solve for y. $\frac{(5y-5)^3}{2} = 4$.
- Given that $x = 2$ is a solution of the polynomial $20x^3 - 10x^2 - 220x + 400 = 0$, find all other solutions.
- Find an equation that has $x = -2$ and $x = 4$ as solutions.
- Find a polynomial of degree 2 with the factors 3, $(x - 7)$, and $(x + 3)$.
- **On Your Own**: If $x = 3$ is a solution of $x^3 - 6x^2 - x = -30$, find all other solutions to the equation.

Rational Equations

Recall:

- Solve. $\frac{1}{6}x + \frac{1}{2} = \frac{1}{3}$

A **rational equation** is an equation that contains a rational expression.

We have already solved linear equations that contained fractions. We found the LCD of all the fractions in the equation and then multiplied both sides of the equation by the LCD to "clear" the fractions.

We will use the same strategy to solve rational equations. The original equation may have a variable in a denominator, so we must be careful that we don't end up with a solution that would make a denominator equal to zero.

Rational Equations

So before we begin solving a rational equation, we examine it first to find the values that would make any denominators zero. That way, when we solve a rational equation we will know if there are any algebraic solutions we must discard. Alternatively, we can check to make sure your possible solutions make sense in our equation.

A solution to an an equation that is equivalent (except for a few values) to a rational equation for which the rational expressions are undefined is called an **extraneous solution to a rational equation**.

Rational Equations

An **extraneous solution to a rational equation** is a solution to an equation which is that is equivalent to the original except for a certain finite number of values that would cause any of the expressions in the original equation to be undefined.

We note any possible extraneous solutions, c, by writing $x \neq c$ next to the equation.

Example: Solve. Note any possible extraneous solutions.

- $\frac{1}{x} + \frac{1}{3} = \frac{5}{6}$

Rational Equations

How to Solve Equations with Rational Expressions

1. Note any value of the variable that would make any denominator zero.
2. Find the least common denominator of all denominators in the equation.
3. Clear the fractions by multiplying both sides of the equation by the LCD.
4. Solve the resulting equation.
5. Check.
    - If any values found in Step 1 are algebraic solutions, discard them.
    - Check any remaining solutions in the original equation.

Note: It is possible that a rational equation has no solutions.

Rational Equations

Example: Solve. Note any possible extraneous solutions.

- $1 - \frac{2}{x} = \frac{15}{x^2}$
- $\frac{2}{x+2} + \frac{4}{x-2} = \frac{x-1}{x^2-4}$
- $\frac{m+11}{m^2-5m+4} = \frac{5}{m-4} - \frac{3}{m-1}$
- $\frac{y}{y+6} = \frac{72}{y^2-36} + 4$
- **On Your Own**: $x + \frac{4}{x} = 4$
- **On Your Own**: $\frac{x}{5x-10} + \frac{4}{x^2-4} = \frac{1}{5}$