Chapters 2.2.4 - 2.2.5: Solving Quadratic Equations Using the Quadratic Formula and Applications of Quadratic Equations

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Solving Quadratic Equations

So far, we have studied various ways to solve quadratic equations of the form $ax^2 + bx + c = 0$ when $a \neq 0$.

- Factoring and the Zero-Product Property
- Square-Root Property
- Completing the Square

Now, we will learn another method for solving quadratic equations - the **Quadratic Formula**.

The Quadratic Formula

The solutions to a **quadratic equation** $ax^2 + bx + c = 0$, where $a \neq 0$ are given by the **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

To use the Quadratic Formula, we substitute the values of a, b and c from the standard form into the expression on the right side of the formula. Then we simplify the expression. The result is the pair of solutions to the quadratic equation.

Solve Using the Quadratic Formula

Solve a Quadratic Equation Using the Quadratic Formula

1. Write the quadratic equation in standard form, $ax^2 + bx + c = 0$. Identify a, b and c. (Be careful about the signs!)
2. Write the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   Then substitute in the values of a, b and c.

3. Simplify. (Remember, you may get a single solution if the trinomial is a perfect square trinimial!)
4. Check the solutions to detect errors. (This step is only to guard against mistakes!)

Remembering the Quadratic Formula

There are multiple mnemonic devices to help you remember the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The following is sung to the tune of Pop Goes the Weasel.
"X equals negative b plus or minus square root b squared minus 4ac all over 2a."

Here is a "story" that may help.
"A negative boy was undecided whether or not he should go to a radical party but his 2 friends who were boys went and there they met 4 amazing chicks and stayed up until 2am."

· negative boy - $(-b)$
· undecided - $(\pm)$
· a radical party - (square root symbol)
· 2 friend boys - (b^2)
· met 4 amazing chicks - (minus $4ac$)
· stayed up until 2am - (division by $2a$)

There are various other mnemonic device online for remembering the Quadratic Formula. Look some up and see what works best for you!

Solve Using the Quadratic Formula

Examples: Solve using the Quadratic Formula. Check your answers!

- $3y^2 - 5y + 2 = 0$

Use the Discriminant to Predict the Number and Type of Solutions of a Quadratic Equation

If $ax^2 + bx + c = 0$ and $a \neq 0$, the quantity

$$b^2 - 4ac$$

is called the **discriminant**. It is the radicand in the quadratic formula.

For a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$,

- If $b^2 - 4ac > 0$, the equation has 2 real solutions.
- If $b^2 - 4ac = 0$, the equation has 1 real solution.
- If $b^2 - 4ac < 0$, the equation has 2 complex solutions.

Examples: Solve using the Quadratic Formula. If there is a radical in the solution, the final answer should have the radical in its simplified form.

- $2x^2 + 9x - 5 = 0$
- $2x^2 + 10x + 11 = 0$
- $5p^2 + 2p + 4 = 0$
- **On Your Own**: $4x^2 - 20x = -25$

Review

Review of Methods: Solve using the indicated method. If there is a radical in the solution, the final answer should have the radical in its simplified form. Complex numbers should be written in standard form as well.

- Solve by factoring and using the Zero-Product Property: $2x^2 = 72 - 10x$
- Solve using the Square-Root Property: $6(x+1)^2 - 48 = 0$
- Solve by Completing the Square: $x^2 + 3x + 1 = 0$
- Solve using the Quadratic Formula: $5x + 2x^2 = -2$

Applications of Quadratic Equations

We solved some applications that are modeled by quadratic equations earlier, when the only method we had to solve them was factoring. Now that we have more methods to solve quadratic equations, we will take another look at applications.

Problem Solving Strategy for Application Problems (Word Problems)

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** all important information and the problem we want to solve (the GOAL). Draw a picture if appropriate.
3. **Name** what we are looking for. Choose a variable to represent that quantity and all relevant quantities.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation using your variables.
5. If necessary, write the equation in **one variable**. If there are multiple variables, use the information form the problem to relate variables to one another and substitute.
   If the equation already has one variable, skip this step.
6. **Solve** the equation using algebra techniques.
7. **Check** the answer in the problem and make sure it makes sense. Make sure your solution solves the problem. (Have you achieved the GOAL?)
8. **Answer** the question with a complete sentence.

Applications of Quadratic Equations

Examples: Round answers to the nearest tenth when appropriate.

- Find two positive integers whose difference is 7 and the sum of their squares is 85.
- An architect is designing a triangular window for a doorway. Due to energy restrictions, the window can only have an area of 120 square feet and the architect wants the base to be 4 feet more than twice the height. Find the base and height of the window.
- The surface area of a cube is 150 cm^2. What is the volume of the cube?
- The sun casts a shadow from a flag pole. The height of the flag pole is three times the length of its shadow. The distance between the end of the shadow and the top of the flag pole is 20 feet. Find the length of the shadow and the length of the flag pole.

Chapter Review

On Your Own: Solve using the indicated method. If there is a radical in the solution, the final answer should have the radical in its simplified form. Complex numbers should be written in standard form as well.

- Solve using the Quadratic Formula: $x^2 - 6x = -5$
- Solve by factoring and using the Zero-Product Property: $2x^3 - 30x^2 = -72x$ (Hint: You should have three solutions!)
- Solve using the Quadratic Formula: $\frac{1}{2}u^2 + \frac{2}{3}u = \frac{1}{3}$ (Hint: It may be helpful to clear fractions first!)
- Solve using the Square-Root Property: $2(x-2)^2 + 3 = 57$
- Solve by Completing the Square: $a^2 + 4a + 9 = 30$
- The distance between opposite corners of a rectangular field is four more than the width of the field. The length of the field is twice its width. Find the distance between the opposite corners. Round to the nearest tenth.