Chapter 2.2.1: Solving Quadratic Equations Using the Zero-Product Property

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Spring 2024

Quadratic Equations

A **solution** to an equation in a variable (lets call it x) is a number that we can substitute in for x that makes the equation true.

Two equations are **equivalent** if they have the same solutions.

A **quadratic equation with one variable**, say x, has polynomials on both sides of the equation and can be written in **standard form**: $Ax^2 + Bx + C = 0$ where A is not zero, i.e., there is an equivalent equation of the form $Ax^2 + Bx + C = 0$.

Expressions versus Equations

A quadratic expression and a quadratic equation are not the same types of objects. Here are some examples.

- Expressions: $2x + 1, 0$
  Linear Equation: $2x + 1 = 0$
- Expressions: $x^2, x - 2$
  Quadratic Equation: $x^2 = x - 2$

Finding solutions to a quadratic equation is, in general, a little more difficult than finding solution to a linear equation. But we know that we can sometimes write a quadratic equation as a product of two linear factors (or two linear expressions and a number). As we will see this can be vary handy!

The zero-product property

The Zero Product Property: If $ab = 0$ then $a = 0$ or $b = 0$ or both.

Examples:

- Solve. $(2x + 3)(3x + 5) = 0$
- Solve. $(x - 5)(x + 2) = 0$

Solve Quadratic Equations by Factoring

The Zero Product Property works very nicely to solve quadratic equations. The quadratic equation must be factored, with zero isolated on one side. So be sure to start with the quadratic equation in **standard form**, $ax^2 + bx + c = 0$. Then factor the expression on the left.

Solve a Quadratic Equation by Factoring

1. Write the quadratic equation in standard form $ax^2 + bx + c = 0$.
2. Factor the quadratic expression.
3. Use the Zero Product Property.
4. Solve the linear equations.
5. Optional: Check answer by substituting each solution separately into the original equation to see if a mistake has been made.

Solve Quadratic Equations by Factoring

Examples: Solve.

- $4q^3 = 25q$
- $x^2 + 5x + 6 = 0$
- $3x^2 = 12x + 63$
- **On Your Own**: $(k + 1)(k - 1) = 8$

Solve Quadratic Equations by Factoring

Examples:

- A rectangular sign has area 30 square feet. The length of the sign is one foot more than the width. Find the length and width of the sign.

- The difference of the squares of two consecutive even integers is 68. What are these numbers?

- In this example, we will use the **Pythagorean Theorem** $(a^2 + b^2 = c^2)$. This formula gives the relation between the legs and the hypotenuse of a right triangle.
  The sides of a right triangle are x, $x + 1$ and $x + 2$ units long. Find the side lengths of the triangle by solving for x.

- **On Your Own**: A rectangular patio has area 180 square feet. The width of the patio is three feet less than the length. Find the length and width of the patio.