Chapter 1.4.5 - 1.4.6: Dividing Radical Expressions and Complex Numbers

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Divide Radical Expressions

Quotient Property of Radical Expressions: If $\sqrt{a}$ and $\sqrt{b}$ are real numbers with $b \neq 0$, then

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}},$$

or in the language of exponents

$$\left(\frac{a}{b}\right)^{\frac{1}{2}} = \frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}}.$$

Examples: Simplify.

- $\dfrac{\sqrt{72x^3}}{\sqrt{162x}}$
- $\dfrac{\sqrt{54x^5y^3}}{\sqrt{3x^2y}}$

Rationalize a One-Term Denominator

Rationalizing the denominator is the process of converting a fraction with a radical in the denominator to an equivalent fraction whose denominator is an integer.

Simplified Radical Expressions: A radical expression is considered **simplified** if there are

- no factors in the radicand have perfect powers of the index
- no fractions in the radicand
- no radicals in the denominator of a fraction

Rationalize a One-Term Denominator

To rationalize a denominator with a square root, we use the property that $(\sqrt{a})^2 = a$. We will use this property to rationalize the denominator in the next examples.

Examples: Simplify.

- $\frac{4}{\sqrt{3}}$
- $\sqrt{\frac{3}{20}}$
- On Your Own: $\frac{3}{\sqrt{6x}}$

Rationalize a Two-Term Denominator

When the denominator of a fraction is a sum or difference with square roots, we use the **Product of Conjugates Pattern** to rationalize the denominator.

$$(a - b)(a + b) = a^2 - b^2$$

Examples: Simplify.

- $\frac{5}{2-\sqrt{3}}$
- $\frac{3}{1-\sqrt{5}}$
- On Your Own: $\frac{x-3}{\sqrt{x}+\sqrt{3}}$

Evaluating the Square Root of a Negative Number

Whenever we have a situation where we have a square root of a negative number we say there is no real number that equals that square root. Since all real numbers squared are positive numbers, there is no real number that equals -1 when squared.

Now we will expand the real numbers to include the square roots of negative numbers.

The **imaginary unit** i is a number whose square is -1, i.e. $i^2 = -1$.

Square Root of a Negative Number

Square Root of a Negative Number: If b is a positive real number, then

$$\sqrt{-b} = \sqrt{b}i.$$

Sometimes we write $\sqrt{-b} = i\sqrt{b}$ to emphasize the i is not under the radical but $\sqrt{b}i$ is standard form.

Examples: Write each expression in terms of i and simplify.

- $\sqrt{-25}$
- $\sqrt{-12}$
- $\sqrt{-7}$

Complex Numbers

Now that we are familiar with the imaginary number i, we can expand the real numbers to include imaginary numbers.

A **complex number** is of the form $a + bi$, where a and b are real numbers.

- The number a is called the **real part** of $a + bi$.
- The number b is called the **real part** of $a + bi$.

A complex number is in **standard form** when written as $a + bi$, where a and b are real numbers.

What happens when $b = 0$? When $b \neq 0$? When $a = 0$?

Add or Subtract Complex Numbers

Adding and subtracting complex numbers is much like adding or subtracting like terms. We add or subtract the real parts and then add or subtract the imaginary parts. Our final result should be in standard form.

Examples: Add.

- $\sqrt{-12} + \sqrt{-27}$
- $\sqrt{-8} + \sqrt{-32}$
- $(4 - 3i) + (5 + 6i)$
- $(8 - 4i) - (2 - i)$

Multiply Complex Numbers

Multiplying complex numbers is also much like multiplying expressions with coefficients and variables.

Examples: Multiply.

- $\sqrt{-36} \cdot \sqrt{-4}$
- $2i(7 - 5i)$
- $4i(5 - 3i)$
- $(3 + \sqrt{-2})(4 - \sqrt{-3})$
- On Your Own: $(3 + 2i)^2$

Complex Conjugate Pairs

A **complex conjugate pair** is of the form $a + bi$, $a - bi$.

Product of Complex Conjugates: If a and b are real numbers, the

$$(a + bi)(a - bi) = a^2 + b^2$$

Examples: Multiply.

- $(8 - 2i)(8 + 2i)$
- $(2 + \sqrt{-1})(2 - \sqrt{-1})$

Divide Complex Numbers

Dividing complex numbers is much like rationalizing a denominator. We want our result to be in standard form with no imaginary numbers in the denominator.

How to Divide Complex Numbers:

1. Write both the numerator and denominator in standard form.
2. Multiply both the numerator and denominator by the complex conjugate of the denominator.
3. Simplify and write the result in standard form.

Examples: Divide.

- $\frac{4+3i}{3-4i}$
- $\frac{-2}{-1+2i}$
- $\frac{5+3i}{4i}$

Section Review

Examples: Write each expression in terms of i (if possible) and simplify.

- $\frac{3}{\sqrt{5}}$
- $\frac{\sqrt{p}-\sqrt{2}}{\sqrt{p}+\sqrt{2}}$
- $(2+7i)+(4-2i)$
- $(-2+\sqrt{-8})(3-\sqrt{-18})$
- $\frac{3+3i}{2i}$