Chapter 1.4.3 - 1.4.4: Rational Exponents and Adding, Subtracting and Multiplying Radical Expressions

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The n^{th} Root

In the previous sections we worked with the simplest radical expressions, i.e. square roots. This can be generalized as follows.

The n^{th} Root: The n^{th} **root** of a is b if $b^n = a$. If n is even, take b to be positive and we write $\sqrt[n]{a} = b$. We call n the **index** and a the **radicand**.

Examples: Evaluate.

- $\sqrt[4]{625}$
- $\sqrt[3]{-27}$
- $\sqrt[5]{-32x^{10}y^5z^{20}}$

Simplify Expressions with $a^{\frac{1}{n}}$

Rational exponents are another way of writing expressions with radicals. When we use rational exponents, we can apply the properties of exponents to simplify expressions.

Rational Exponents: If $\sqrt[n]{a}$ is a real number and $n \geq 2$, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}.$$

The denominator of the rational exponent is the index of the radical.

Examples: Write as a radical expression.

- $z^{\frac{1}{5}}$
- $x^{\frac{1}{3}}$
- $a^{\frac{1}{8}}$

Rational Exponents

Examples: Write with a rational exponent.

- $\sqrt{5x}$
- $\sqrt[4]{3x}$

For any positive integres m and n,

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m \quad \text{and} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}.$$

We can use either form to simplify an expression but we usually take the root first—that way we keep the numbers in the radicand smaller, before raising it to the power indicated.

Rational Exponents

Examples: Write with a rational exponent.

- $\sqrt{y^3}$
- $\left(\sqrt[3]{2x}\right)^4$
- $\sqrt[5]{\left(\frac{3a}{4b}\right)^3}$

Examples: Simplify.

- $4^{\frac{3}{2}}$
- $27^{-\frac{2}{3}}$
- On Your Own: $-16^{-\frac{5}{4}}$
- On Your Own: $(-16)^{-\frac{5}{4}}$

Properties of Exponents with Rational Exponents

The same properties of exponents that we have already used for integers also apply to rational exponents. Recall the following properties.

Properties of Exponents

If a and b are real numbers and m and n are rational numbers, then

Product Property

$$a^m \cdot a^n = a^{m+n}$$

Power Property

$$(a^m)^n = a^{mn}$$

Product to a Power

$$(ab)^m = a^m b^m$$

Quotient Property

$$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$$

Zero Exponent Definition

$$a^0 = 1, a \neq 0$$

Quotient to a Power Property

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$$

Negative Exponent Property

$$a^{-n} = \frac{1}{a^n}, a \neq 0$$

Properties of Exponents with Rational Exponents

Examples: Simplify. Do not leave negative exponents in your final answers.

- $x^{\frac{1}{2}} \cdot x^{\frac{5}{6}}$
- $\left(z^9\right)^{\frac{2}{3}}$
- $\frac{x^{\frac{1}{3}}}{x^{\frac{5}{3}}}$
- On Your Own: $\left(\frac{x^{-1/2}}{y^{-3/5}}\right)^{20}$

Add and Subtract Radical Expressions

Adding radical expressions with the same index and the same radicand is just like adding like terms. We call radicals with the same index and the same radicand like radicals to remind us they work the same as like terms.

Like radicals are radical expressions with the same index and the same radicand.

We add and subtract like radicals in the same way we add and subtract like terms.

Examples: Simplify.

- $3\sqrt{8} + 7\sqrt{8}$
- $2\sqrt{2y} - 10\sqrt{2y}$

Add and Subtract Radical Expressions

Remember that we always simplify radicals by removing the largest factor from the radicand that is a power of the index.

Examples: Simplify.

- $\sqrt{20} + 3\sqrt{5}$
- $\sqrt{18} - 6\sqrt{2}$
- On Your Own: $\sqrt{27m^3} - \sqrt{48m^3}$

Multiply Radical Expressions

Product Property of Roots: For any real numbers, $\sqrt{a}$ and $\sqrt{b}$, we have

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \quad \text{and} \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{ab}.$$

Examples: Simplify.

- $(6\sqrt{2})(3\sqrt{10})$
- $(5\sqrt{8p^3})(2\sqrt{2p})$
- On Your Own: $(\sqrt{2x^7})(3\sqrt{72x^4})$

Use Polynomial Multiplication to Multiply Radical Expressions

In the next a few examples, we will use the Distributive Property to multiply expressions with radicals. First we will distribute and then simplify the radicals when possible.

Examples: Simplify.

- $\sqrt{6}(\sqrt{2} + \sqrt{18})$
- $\sqrt{8}(2 - 5\sqrt{8})$

Use Polynomial Multiplication to Multiply Radical Expressions

When we worked with polynomials, we multiplied binomials by binomials using FOIL. We will do something similar in the following examples.

Examples: Simplify.

- $(6 - 3\sqrt{7})(3 + 4\sqrt{7})$
- $(10 + \sqrt{2})^2$
- $(5 - 2\sqrt{3})(5 + 2\sqrt{3})$

Section Review

Examples: Simplify. Do not use negative exponents in your final answers.

- $3\sqrt{80} - 5\sqrt{45}$
- $5p\sqrt{72p^2} + 2p^2\sqrt{8}$
- $z^{-4/7}z^{-6/7}$
- $-81^{3/4}$
- $(2\sqrt{a} - \sqrt{ab})^2$
- $\sqrt{5}(\sqrt{15} - \sqrt{3})$