Chapter 1.4.1 - 1.4.2: Radical Expressions and Simplifying Radical Expressions

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Simplify Expressions with Roots

Square

- If $n^2 = m$, then m is the square of n.

Square Root

- If $n^2 = m$, then n is the square root of m.

In words, a square root of m is a number whose square is m.

Notice, every positive number has two square roots — one positive and one negative.

$$(3)^2 = 9 \qquad\qquad (-3)^2 = 9$$

We use a *radical sign*, $\sqrt{m}$, which denotes the positive square root of the *radicand* m. The non-negative square root is also called the **principal square root**. This is the square root approximated by using the root symbol of your calculator!

Simplify Expressions with Roots

Square Root: We read $\sqrt{m}$ as "the square root of m." If $n^2 = m$, then $n = \sqrt{m}$, for $n \geq 0$.

We use the radical sign for the square root of 0 as well. Since $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root.

Rule: We know that every positive number has two square roots and the radical sign indicates the positive one. If we want to find the negative square root of a number, we place a negative in front of the radical sign.

Examples: Simplify.

- $\sqrt{144}$
- $-\sqrt{49}$
- $\sqrt{-25}$

Simplify Expressions with Roots

Any positive number squared is positive. Any negative number squared is positive. There is no real number equal to $\sqrt{-25}$.

Properties of $\sqrt{a}$: When

- $a \geq 0$, then $\sqrt{a}$ is a real number.
- $a < 0$, then $\sqrt{a}$ is a not real number.

The principle square root is always positive.

$$\sqrt{(-4)^2} = 4 \quad \text{and} \quad \sqrt{4^2} = 4$$

We say $\sqrt{a^2} = |a|$. The absolute value guarantees the principal root is positive. We must use the absolute value signs when we take a square root of an expression with a variable in the radical.

Absolute value function: $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

Square Roots of Higher Powers

The power property of exponents says $(a^m)^n = a^{m \cdot n}$.

$$(a^m)^2 = a^{2m}$$

Looking now at the square root.

$$\sqrt{a^{2m}} = \sqrt{(a^m)^2}$$

Since $\sqrt{x^2} = |x|$ (from last slide),

$$\sqrt{(a^m)^2} = |a^m|.$$

Examples: Simplify.

- $\sqrt{y^{18}}$
- $\sqrt{16x^8}$

Section Review

Examples: Simplify.

- $\sqrt{100}$
- $\sqrt{(-100)^2}$
- $\sqrt{36x^2y^2}$
- $\sqrt{121a^6b^8}$

Simplifying Radical Expressions

We will simplify radical expressions in a way similar to how we simplified fractions.

For non-negative integers a and m, a **radical expression** $\sqrt{a}$ is considered simplified if a has no factors of the form m^2.

Examples: Are the following radical expressions simplified?

- $\sqrt{5}$
- $\sqrt{12}$

The Product Property of Roots

To simplify radical expressions, we will also use some properties of roots.

Product Property of Roots: If $\sqrt{a}$ and $\sqrt{b}$ are real numbers, then

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}.$$

Simplify a Radical Expression Using the Product Property

1. Find the largest factor in the radicand that is a perfect power of the index (for now, the index is 2). Rewrite the radicand as a product of two factors, using that factor.
2. Use the product rule to rewrite the radical as the product of two radicals.
3. Simplify the root of the perfect power.

The Product Property of Roots

Examples: Simplify. Don't forget to use the absolute value signs when taking an even root of an expression with a variable in the radical.

- $\sqrt{500}$
- $\sqrt{x^3}$
- $\sqrt{75w^9}$

The Quotient Property of Radical Expressions

Let's consider roots of rational expressions. Recall, $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$.
Examples: Simplify.

- $\sqrt{\frac{45}{80}}$
- $\sqrt{\frac{m^6}{m^4}}$

The Quotient Property of Radical Expressions: If $\sqrt{a}$ and $\sqrt{b}$ are real numbers and $b \neq 0$, then

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}.$$

Simplify a Square Root Using the Quotient Property

1. Simplify the fraction in the radicand, if possible.
2. Use the Quotient Property to rewrite the radical as the quotient of two radicals.
3. Simplify the radicals in the numerator and the denominator.

The Quotient Property of Radical Expressions

When the radical expression in the numerator or denominator cannot be simplified, we can use the Quotient Property to combine the numerator and denominator into one radical.

Examples: Simplify.

- $\sqrt{\frac{24p^3}{49}}$
- $\sqrt{\frac{18p^5q^7}{32pq^2}}$
- $\frac{\sqrt{48a^7}}{\sqrt{3a}}$
- $\frac{\sqrt{128m^9}}{\sqrt{2m}}$

Section Review

Examples: Simplify.

- $\sqrt{\frac{49}{25}}$
- $\sqrt{180}$
- $\sqrt{x^7}$
- $\sqrt{9a^2b^3}$
- $\sqrt{\frac{3x}{192x^{13}}}$
- $\frac{\sqrt{125x^{11}y^5}}{\sqrt{5xy}}$