Chapter 1.3.3 - 1.3.4: Adding and Subtracting Rational Expressions and Complex Rational Expressions

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Dr. Davie

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Add and Subtract Rational Expressions with a Common Denominator

The first step we take when we add numerical fractions is to check if they have a common denominator. If they do, we add the numerators and place the sum over the common denominator. If they do not have a common denominator, we find one before we add.

It is the same with rational expressions. To add rational expressions, they must have a common denominator. When the denominators are the same, we add the numerators and place the sum over the common denominator.

Add and Subtract Rational Expressions with a Common Denominator

Rational Expression Addition and Subtraction: If p, q and r are polynomials where $r \neq 0$, the

$$\frac{p}{r} + \frac{q}{r} = \frac{p+q}{r} \qquad \text{and} \qquad \frac{p}{r} - \frac{q}{r} = \frac{p-q}{r}.$$

Remember:

- We always simplify rational expressions. Factor whenever possible.
- We do not allow values that would make the denominator zero.

Add and Subtract Rational Expressions with a Common Denominator

Examples: Add, subtract and simplify the rational expressions. What values of x should we exclude in each example?

- $\frac{9x+14}{x+7} + \frac{x^2}{x+7}$

- $\frac{x^2+8x}{x+5} + \frac{15}{x+5}$

- $\frac{5x^2-7x+3}{x^2-3x+18} - \frac{4x^2+x-9}{x^2-3x+18}$

- $\frac{7x+2}{x^2-2x-24} - \frac{3x-1}{x^2-2x-24}$

Add and Subtract Rational Expressions Whose Denominators are Opposites

When the denominators of two rational expressions are opposites, it is easy to get a common denominator. We just have to multiply one of the fractions by $\frac{-1}{-1}$.

Be careful with the signs as we work with the opposites when the fractions are being subtracted.

Examples: Add, subtract and simplify the rational expressions. What values of x (y or m) should we exclude in each example?

- $\frac{5}{x-1} + \frac{3}{1-x}$
- $\frac{m^2-6m}{m^2-1} - \frac{3m+2}{1-m^2}$
- On Your Own: $\frac{y^2-5y}{y^2-4} - \frac{6y-6}{4-y^2}$

Find the Least Common Denominator of Rational Expressions

Find the Least Common Denominator (LCD) of Rational Expressions

1. Factor each denominator completely.
2. List the factors of each denominator. Match factors vertically when possible.
3. Bring down the columns by including all factors, but do not include common factors twice.
4. Write the LCD as the product of the factors.

Find the Least Common Denominator of Rational Expressions

Examples: Find the LCD for the expressions and rewrite them as equivalent rational expressions with the lowest common denominator.

- $\frac{2}{x+5}$ and $\frac{x}{3}$
- $\frac{8}{x^2-2x-3}$ and $\frac{3x}{x^2+4x+3}$
- On Your Own: $\frac{2}{x^2-x-12}$ and $\frac{1}{x^2-16}$

Add and Subtract Rational Expressions with Unlike Denominators

Now we have all the steps we need to add or subtract rational expressions with unlike denominators.

Examples: Add, subtract and simplify the rational expressions. What values of x should we exclude in each example?

- $\frac{3}{x-3} + \frac{2}{x-2}$
- $\frac{2}{x-2} - \frac{9}{x^2-4}$

Add or Subtract Rational Expressions

Add or Subtract Rational Expressions

1. Determine if the expressions have a common denominator.
   - **YES** - Go to Step 2.
   - **NO** - Rewrite each rational expression with the LCD.
     - Find the LCD.
     - Rewrite each rational expression as an equivalent rational expression with the LCD.

2. Add or subtract the rational expressions.

3. Simplify, if possible.

Examples: Add, subtract and simplify.

- $\frac{1}{m^2-m-2} + \frac{5m}{m^2+3m+2}$
- $\frac{2n}{n^2-3n-10} + \frac{6}{n^2+5n+6}$

Section Review

Examples: Add, subtract and simplify. Be careful with your signs!

- $\frac{8y}{y^2-16} + \frac{4}{4-y}$
- $\frac{-3n-9}{n^2+n-6} - \frac{n+3}{2-n}$
- $\frac{3x-1}{x^2-5x-6} - \frac{2}{6-x}$
- $\frac{4}{a^2+6a+5} - \frac{3}{a^2+7a+10}$

Complex Rational Expressions

A **complex rational expression** is a rational expression in which the numerator and/or the denominator contains a rational expression.

$$\frac{\frac{3x}{4} + \frac{x}{2}}{\frac{5x}{8}} \qquad \text{and} \qquad \frac{\frac{x+4}{2}}{\frac{xy}{7+x}}$$

How to Simplify a Complex Rational Expression by Writing It as Division

1. Rewrite the complex rational expression as a division problem.
2. Divide the expressions.

Complex Rational Expressions

Examples: Simplify the complex rational expressions.

- $\dfrac{\frac{6}{x-4}}{\frac{3}{x^2-16}}$

- $\dfrac{\frac{1}{x^2-7x+12}}{\frac{2}{x-4}}$

Complex Rational Expressions

Fraction bars act as grouping symbols. So to follow the Order of Operations, we simplify the numerator and denominator as much as possible before we can do the division.

Examples: Simplify the complex rational expressions.

- $\dfrac{\frac{1}{3}+\frac{1}{6}}{\frac{1}{2}-\frac{1}{3}}$

- $\dfrac{b-\frac{3b}{b+5}}{\frac{2}{b+5}+\frac{1}{b-5}}$

Section Review

Examples: Simplify the complex rational expressions.

- $\dfrac{\frac{1}{x} + \frac{1}{y}}{\frac{x}{y} - \frac{y}{x}}$

- $\dfrac{\frac{3y}{y+4}}{\frac{5}{7y+28}}$

- $\dfrac{\frac{2x}{x+8} - 1}{\frac{x^2 - 64}{x+4}}$

- $\dfrac{\frac{3}{a} - \frac{4}{a+1}}{\frac{3}{a+1} - \frac{4}{a}}$