

Chapter 1.3.3: Adding and Subtracting Rational Expressions

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Add and Subtract Rational Expressions with a Common Denominator

The first step we take when we add numerical fractions is to check if they have a common denominator. If they do, we add the numerators and place the sum over the common denominator. If they do not have a common denominator, we find one before we add.

It is the same with rational expressions. To add rational expressions, they must have a common denominator. When the denominators are the same, we add the numerators and place the sum over the common denominator.

Add and Subtract Rational Expressions with a Common Denominator

Rational Expression Addition and Subtraction: If p , q and r are polynomials where $r \neq 0$, the

$$\frac{p}{r} + \frac{q}{r} = \frac{p+q}{r} \quad \text{and} \quad \frac{p}{r} - \frac{q}{r} = \frac{p-q}{r}.$$

Remember:

- We always simplify rational expressions. Factor whenever possible.
- We do not allow values that would make the denominator zero.

Add and Subtract Rational Expressions with a Common Denominator

Examples: Add, subtract and simplify the rational expressions. What values of x should we exclude in each example?

$$\bullet \frac{9x+14}{x+7} + \frac{x^2}{x+7}$$

$$\bullet \frac{x^2+8x}{x+5} + \frac{15}{x+5}$$

$$\bullet \frac{5x^2-7x+3}{x^2-3x+18} - \frac{4x^2+x-9}{x^2-3x+18}$$

$$\bullet \frac{7x+2}{x^2-2x-24} - \frac{3x-1}{x^2-2x-24}$$

Add and Subtract Rational Expressions Whose Denominators are Opposites

When the denominators of two rational expressions are opposites, it is easy to get a common denominator. We just have to multiply one of the fractions by $\frac{-1}{-1}$.

Be careful with the signs as we work with the opposites when the fractions are being subtracted.

Examples: Add, subtract and simplify the rational expressions. What values of x (y or m) should we exclude in each example?

- $\frac{5}{x-1} + \frac{3}{1-x}$

- $\frac{m^2-6m}{m^2-1} - \frac{3m+2}{1-m^2}$

- On Your Own: $\frac{y^2-5y}{y^2-4} - \frac{6y-6}{4-y^2}$

Find the Least Common Denominator of Rational Expressions

Find the Least Common Denominator (LCD) of Rational Expressions

- 1 Factor each denominator completely.
- 2 List the factors of each denominator. Match factors vertically when possible.
- 3 Bring down the columns by including all factors, but do not include common factors twice.
- 4 Write the LCD as the product of the factors.

Find the Least Common Denominator of Rational Expressions

Examples: Find the LCD for the expressions and rewrite them as equivalent rational expressions with the lowest common denominator.

- $\frac{8}{x^2-2x-3}$ and $\frac{3x}{x^2+4x+3}$
- $\frac{2}{x^2-x-12}$ and $\frac{1}{x^2-16}$
- On Your Own: $\frac{3x}{x^2-3x+10}$ and $\frac{5}{x^2+3x+2}$

Add and Subtract Rational Expressions with Unlike Denominators

Now we have all the steps we need to add or subtract rational expressions with unlike denominators.

Examples: Add, subtract and simplify the rational expressions. What values of x should we exclude in each example?

- $\frac{3}{x-3} + \frac{2}{x-2}$

- $\frac{2}{x-2} + \frac{5}{x+3}$

Add or Subtract Rational Expressions

Add or Subtract Rational Expressions

- 1 Determine if the expressions have a common denominator.
 - **YES** - Go to Step 2.
 - **NO** - Rewrite each rational expression with the LCD.
 - Find the LCD.
 - Rewrite each rational expression as an equivalent rational expression with the LCD.
- 2 Add or subtract the rational expressions.
- 3 Simplify, if possible.

Examples: Add, subtract and simplify.

- $\frac{1}{m^2-m-2} + \frac{5m}{m^2+3m+2}$
- $\frac{2n}{n^2-3n-10} + \frac{6}{n^2+5n+6}$

Section Review

Examples: Add, subtract and simplify. Be careful with your signs!

$$\bullet \frac{8y}{y^2-16} + \frac{4}{4-y}$$

$$\bullet \frac{-3n-9}{n^2+n-6} - \frac{n+3}{2-n}$$

$$\bullet \frac{3x-1}{x^2-5x-6} - \frac{2}{6-x}$$

$$\bullet \frac{4}{a^2+6a+5} - \frac{3}{a^2+7a+10}$$

$$\bullet \frac{3}{b^2-4b-5} - \frac{2}{b^2-6b+5}$$