Chapter 1.3.2: Simplifying, Multiplying and Dividing Rational Expressions

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Rational Expressions

A **rational expression** is an expression of the form $\frac{p}{q}$, where p and q are polynomials and $q \neq 0$.

Here are some examples of rational expressions:

$$-\frac{21}{47} \qquad \frac{3x}{5y} \qquad \frac{4x+1}{x^2-9} \qquad \frac{4x^2+6x+2}{2x-8}$$

We will do the same operations with rational expressions that we did with fractions. We will simplify, add, subtract, multiply, divide and use them in applications.

Simplify Rational Expressions

We have already been reducing fractions. When c and b are not zero,

$$\frac{ca}{cb} = \frac{a}{b}.$$

The process of replacing $\frac{ca}{cb}$ by the equivalent expression $\frac{a}{b}$ is called reducing the fraction. It is commonly also called **'canceling'** the common factor c.

Simplified Rational Expression: A rational expression is considered **simplified** if there are no common factors other than 1 and 1 in its numerator and denominator. There should be no more "-"s than necessary.

Which of the following rational expressions is simplified?

$$\frac{x+2}{x+3} \qquad \text{or} \qquad \frac{2x}{3x}$$

Equivalent Fractions Property

Recall, we use the Equivalent Fractions Property to simplify numerical fractions.

Equivalent Fractions Property (Reducing Fractions): If a, b and c are numbers where $b \neq 0, c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}.$$

To simplify rational expressions, we first write the numerator and denominator in factored form. Then we remove the common factors using the Equivalent Fractions Property.

Example: Reduce the following expressions by removing the common factors when possible.

(a) $\frac{2 \cdot 2 \cdot 5 \cdot 11}{2 \cdot 3 \cdot 11}$ (b) $\frac{3x(x+4)}{5(x+4)}$ (c) $\frac{x+5}{x}$

Simplifying Rational Expressions: Monomials over Monomials

We have already discussed dividing monomials using the exponent property for division. Note that the result when you divide monomials is not necessarily a polynomial!

When we divide monomials with more than one variable, it is often helpful to write one fraction for each variable.

Example: Simplify the rational expressions.

- $\frac{54a^2b^3}{-6ab^5}$
- On Your Own: $\frac{14x^7y^12}{21x^11y^6}$

Simplifying Rational Expressions: Polynomials over Monomials

We have also discussed dividing monomials by polynomials. To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example: Simplify the rational expressions.

- $\frac{18x^3y-36xy^2}{-3xy}$
- $\frac{32a^7b^3-16a^5b^4}{-8a^2b}$
- On Your Own: $\frac{-42a^8b^4-36a^6b^5}{-6a^3b^3}$

Simplifying Rational Expressions: Polynomials over Polynomials

Now we will consider dividing polynomials by other polynomials, or simplifying rational expressions with polynomials in both the numerator and denominator. We use the following steps to **simplify rational expressions**:

1. Factor the numerator and denominator completely.
2. Simplify by noting common factors.

Usually, we leave the simplified rational expression in factored form. This way, it is easy to check that we have removed all the common factors.

Every time we write a rational expression, we should make a statement disallowing values that would make a denominator zero. However, to let us focus on the work at hand, we will omit writing it in the examples.

Simplifying Rational Expressions: Polynomials over Polynomials

Example: Simplify the rational expressions.

- $\frac{3a^2-12ab+12b^2}{6a^2-24b^2}$
- $\frac{2x^2+14x+24}{2x+6}$

Simplifying Rational Expressions: Polynomials over Polynomials

Opposites in a Rational Expression: The **opposite** of $a - b$ is $b - a$. When $a \neq b$,

$$\frac{a-b}{b-a} = -1.$$

An expression and its opposite divide to -1.
NOTE: Be careful not to treat $a + b$ and $b + a$ as opposites!!!!!

Example: Simplify the rational expressions.

- $\frac{x^2-4x-32}{64-x^2}$
- On Your Own: $\frac{x^2-2x+1}{1-x}$

Multiplying Rational Expressions

If p, q, r, s are polynomials where $q \neq 0$, $r \neq 0$, then

$$\frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs}.$$

The steps for **multiplying rational expressions** are as follows.

1. Factor each numerator and denominator completely.
2. Multiply the numerators and denominators.
3. Simplify by dividing out the common factors.

Example: Simplify.

- $\frac{2x}{x^2-7x+12} \cdot \frac{x^2-9}{6x^2}$
- $\frac{3a^2-8a-3}{a^2-25} \cdot \frac{a^2+10a+26}{3a^2-14a-5}$
- On Your Own: $\frac{4b^2+7b-2}{1-b^2} \cdot \frac{b^2-2b+1}{4b^2+15b-4}$

Dividing Rational Expressions

If p, q, r, s are polynomials where $q \neq 0$, $r \neq 0$, $s \neq 0$, then

$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \cdot \frac{s}{r}.$$

The steps for **multiplying rational expressions** are as follows.

1. Rewrite the division as the product of the first rational expression and the reciprocal of the second.
2. Factor the numerators and denominators completely.
3. Multiply the numerators and denominators.
4. Simplify by dividing out the common factors.

Example: Simplify.

- $\frac{6x^2-7x+2}{4x-8} \div \frac{2x^2-7x+2}{x^2-5x+6}$
- $\frac{3x^2+7x+2}{4x+24} \div \frac{3x^2-14x-5}{x^2+x-30}$
- On Your Own: $\frac{y^2-36}{2y^2+11y-6} \div \frac{2y^3-2y-60}{8y-4}$

Section Review

Simplify.

- Reduce. $\frac{3x^2-30x+48}{x^2-4x-32}$
- Simplify. $\frac{5p^2}{p^2-5p-36} \cdot \frac{p^2-16}{10p}$
- Simplify. $\frac{a^2+8a}{a^3-7a^2} \cdot \frac{14a-98}{2a+16}$
- Simplify. $\frac{2x+6}{4x+20} \div \frac{xy+3x+4y+12}{x^2+8x+16}$
- Simplify. $\frac{2x^2+4x-6}{2x^2-6x+4} \div \frac{2x^2+14x+20}{x^2-4}$