Chapter 1.2.7: Factoring Trinomials

MAT 1275CO
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Factoring Trinomials of the Form $x^2 + bx + c$

You have already learned how to multiply binomials using FOIL. Now you'll need to "undo" this multiplication. To factor the trinomial means to start with the product, and end with the factors.

To figure out how we would factor a **trinomial** of the form $x^2 + bx + c$, let's start with two general binomials of the form $(x + m)$ and $(x + n)$.

Let's FOIL $(x + m)(x + n)$ to get an expression that looks like $x^2 + bx + c$.

Factoring Trinomials of the Form $x^2 + bx + c$

Procedure for Factoring Trinomials of the Form $x^2 + bx + c$:

1. Write the factors as two binomials with first terms x. $(x + \_)(x + \_)$
2. Find two numbers m and n that
   - multiply to c: $mn = c$
   - add to b: $m + n = b$
3. Use m and n as the last terms of the factors. $(x + m)(x + n)$
4. Check by multiplying the factors.

Examples: Factor the following trinomial.

- $x^2 + 9x + 20$

Factoring Trinomials of the Form $x^2 + bx + c$

Strategy for factoring trinomials of the form $x^2 + bx + c$

When we factor a trinomial, we look at the signs of its terms first to determine the signs of the binomial factors.

$x^2 + bx + c$

$= (x + m)(x + n)$

b positive	b negative
m, n positive	m, n negative
When c is positive, m and n have the same sign.	
For example, $x^2 + 5x + 6 = (x + 2)(x + 3)$.	For example, $x^2 - 6x + 8 = (x - 4)(x - 2)$.
When c is negative, m and n have the opposite sign.	
For example, $x^2 + x - 12 = (x + 4)(x - 3)$.	For example, $x^2 - 2x - 15 = (x - 5)(x + 3)$.

Notice that, in the case when m and n have opposite signs, the sign of the one with the larger absolute value matches the sign of b.

Factoring Trinomials of the Form $x^2 + bx + c$

Let's continue to practice the method we just developed to factor trinomials of the form $x^2 + bx + c$.

Examples: Factor the following trinomials.

- $x^2 - 11x + 28$
- $18 + s^2 - 9s$
- On Your Own: $2x + x^2 - 48$

Factoring Trinomials of the Form $x^2 + bxy + cy^2$

When we factor trinomials, we must have the terms written in descending order—in order from highest degree to lowest degree. In the last two examples, we had to rearrange the trinomial first.

Sometimes you'll need to factor trinomials of the form $x^2 + bxy + cy^2$ with two variables, such as $x^2 + 12xy + 36y^2$. The first term, x^2, is the product of the first terms of the binomial factors. The y^2 in the last term means that the second terms of the binomial factors must each contain y.

Examples: Factor the following trinomials.

- $x^2 + 12xy + 36y^2$
- $r^2 - 8rs - 9s^2$

Finding Prime Trinomials

Some trinomials are prime. The only way to be certain a trinomial is prime is to list all the possibilities and show that none of them work.

Examples: Factor the following trinomials.

- $u^2 - 9uv - 12v^2$
- On Your Own: $x^2 + 13x + 7$

Factoring Trinomials of the Form $ax^2 + bx + c$ Using Trial and Error

What happens when the leading coefficient of the polynomial is not 1? We can use trial an error to factor polynomials of this form.

Procedure for Factoring Trinomials of the Form $ax^2 + bx + c$ Using Trial and Error

- Write the trinomial in descending order of degrees as needed.
- Factor any GCF.
- Find all the factor pairs of the first term.
- Find all the factor pairs of the third term.
- Test all the possible combinations of the factors until the correct product is found.
- Check by multiplying.

Factoring Trinomials of the Form $ax^2 + bx + c$ Using Trial and Error

Examples: Factor the following trinomials using trial and error.

- $8a^2 + 20a + 12$
- $4b^2 + 5b + 1$

Factoring Trinomials of the Form $ax^2 + bx + c$ Using the "ac" Method

Another way to factor trinomials of the form $ax^2 + bx + c$ is the "ac" method.

Procedure for Factoring Trinomials of the Form $ax^2 + bx + c$ Using the "ac" Method

1. Factor any GCF.
2. Find the product ac.
3. Find two numbers m and n that
   - multiply to ac: $mn = ac$
   - add to b: $m + n = b$
4. Split the middle term using m and n. $ax^2 + mx + nx + c$
5. Factor by grouping.
6. Check by multiplying the factors.

Factoring Trinomials of the Form $ax^2 + bx + c$ Using the "ac" Method

Examples: Factor the following trinomials using the "ac" method.

- $10a^2 - 55a + 70$
- $16x^2 - 32x + 12$
- On Your Own: $4x^2 + 8x + 3$

Factoring Using Substitution

Sometimes a trinomial does not appear to be in the $ax^2 + bx + c$ form. However, we can often make a thoughtful substitution that will allow us to make it fit the desired form. This is called **factoring by substitution**. It is standard to use u for the substitution. $(au^2 + bu + c)$

Examples: Factor the following trinomials using substitution.

- $10a^2 - 55a + 70$
- $16x^2 - 32x + 12$
- On Your Own: $4x^2 + 8x + 3$

Section Review

Try the following problems on your own! Check your answers when possible.

- Factor. $-7n + 12 + n^2$
- Factor. $m^2 - 13mn + 12n^2$
- Factor. $x^2 - 7x + 8$
- Factor. $12b^2 - 26b + 10$
- Factor. $x^8 + 4x^4 + 4$