Chapter 1.2.6: The Greatest Common Factor and Factoring by Grouping

MAT 1275CO
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Introduction to Factoring

We have already seen that we may reduce fractions if the numerator and denominator have the same factor.

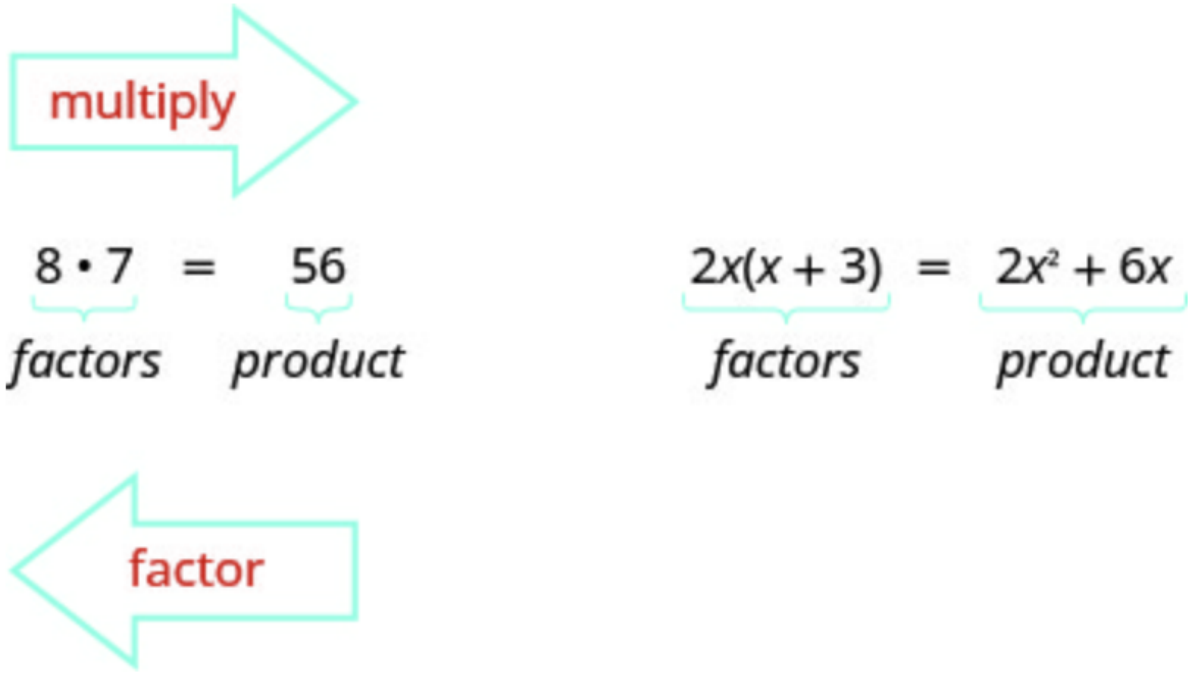
$$\frac{15}{6} = \frac{5 \cdot 3}{2 \cdot 3} = \frac{5}{2}$$

$$\frac{(x-2)(3x-4)}{2(3x-4)} = \frac{x-2}{2} = \frac{x}{2} - \frac{2}{2} = \frac{x}{2} - 1$$

Writing a polynomial as a product of other polynomials (of smaller degree) is the key to reducing fractions. Writing polynomials in such a way is called **factoring**.

Finding the Greatest Common Factor of a Polynomial

Earlier we multiplied factors together to get a product. Now, we will reverse this process; we will start with a product and then break it down into its factors. Splitting a product into factors is called **factoring**.



Finding the Greatest Common Factor of a Polynomial

1. If we write a polynomial P as a product of polynomials, we say that we have **factored** P.
2. A polynomial F is a factor of P if we can write P as $P = F \cdot G$ for some polynomial G.
3. The **greatest common factor (GCF) of two or more monomials** is a monomial F that satisfies the following conditions:
   - F is a factor of all the monomials, that is, F is a common factor, and
   - any other common factor of all the monomials is a factor of F.
4. The **greatest common factor (GCF) of a polynomial** is the GCF of its terms.

While we say the greatest common factor, there are actually two: one with a positive coefficient and one with a negative coefficient. We use "factor" as both a noun and a verb.

Finding the Greatest Common Factor of a Polynomial

We summarize the steps we use to find the greatest common factor.

1. Factor each coefficient into primes. Write all variables with exponents in expanded form.
2. List all factors—matching common factors in a column. In each column, circle the common factors (this is suggested as a way of 'book-keeping').
3. Collect the factors (including repeats) that all polynomials share.
4. Multiply the factors.

Finding the Greatest Common Factor of a Polynomial

Examples:

- Find the greatest common factor of $21x^3, 9x^2$, and $15x$.
- Find the greatest common factor of $25m^4, 35m^3$, and $20m^2$.

Factoring the Greatest Common Factor from a Polynomial

It is sometimes useful to represent a number as a product of factors. To do this we apply the Distributive Property "in reverse."

Distributive Property: If a, b and c are real numbers then

$$a(b+c) = ab + ac$$

and

$$ab + ac = a(b+c).$$

The form on the left is used to multiply. The form on the right is used to factor.

Factoring the Greatest Common Factor from a Polynomial

The procedure for factoring the GCF from a polynomial is as follows.

1. Find the GCF of all the terms of the polynomial.
2. Rewrite each term as a product using the GCF.
3. Use the "reverse" Distributive Property to factor the polynomial.
4. Check by multiplying the factors.

Factoring the Greatest Common Factor from a Polynomial

Examples:Factor out the GCF.

- $5x^3 - 25x^2$
- $6y^3 - 15y^2 + 12y$
- On Your Own: $8x^3y - 20x^2y^2 + 12xy^3$

Factoring the Greatest Common Factor from a Polynomial

We can extend the idea of factoring out the GCF to a binomial.

Examples:Factor out the common factor of the two terms and check your answer.

- $2x(x-5)-5(x-5)$
- $8n(n+2)+4(n+2)$

Factoring by Grouping

Sometimes there is no common factor of all the terms of a polynomial. When there are four terms we separate the polynomial into two parts with two terms in each part. Then look for the GCF in each part. If the polynomial can be factored, you will find a common factor emerges from both parts. **Not all polynomials can be factored.**

Factoring by Grouping

- Group terms with common factors.
- Factor out the common factor in each group.
- Factor the common factor from the polynomial.
- Check by multiplying the factors.

Factoring by Grouping

Examples:Factor by grouping and check your answer.

- $xy + 8y + 3x + 24$
- $x^2 + 2x - 5x - 10$
- $ab + 7b + 8a + 56$

Section Review

Try the following problems on your own! Check your answers when possible.

- Factor. $36x^5 - 6x^3$
- Factor. $6x(y-8) - 2(y-8)$
- Factor by grouping. $-18xy + 21x - 30y + 35$
- Factor by grouping. $10x^2 + 5x - 4x - 2$