Chapter 1.2.4: Powers of Monomials and Binomials

MAT 1275CO

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Powers of Monomials

Recall the following property.

Product to a Power Property for Integer Exponents: $(ab)^m = a^m b^m$

Examples: Simplify.

- $(2x)^5$
- $(-3xy^4)^3$
- On Your Own: $(-4x^2 y^3 z)^2$

The Binomial Theorem

We will now consider powers of binomial expressions like $(x + y)^6$ and $(4x - 2)^{10}$.

$$(x + y)^n = a_0 x^n + a_1 x^{n-1} y + a_2 x^{n-2} y^2 + ... + a_{n-1} x y^{n-1} + a_n y^n,$$

where the coefficients come from the nth row (counting from 0 of the Pascal's triangle):

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10   10   5   1
    1   6   15   20   15   6   1
  1   7   21   35   35   21   7   1
```

Note: To get from one row to the next you add the two numbers above (or 1 in the case of the first and last number).

$$\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1
\end{array}$$

Examples:

- Find the coefficient of x^3 in $(2x - 3)^4$.
- Find the coefficient of xy^2 in $(x - 3y)^3$.