Chapter 1.2.3: Multiplying Polynomials

MAT 1275CO
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Multiplying Monomials

Since monomials are algebraic expressions, we can use the properties of exponents to multiply monomials. Recall the following property.

Product property for exponents: $a^m a^n = a^{m+n}$

Examples: Multiply.

- $(3xy^2)(5x^4y)$
- $\left(\frac{5}{6}x^7y^{-3}\right)\left(12xy^5z^2\right)$

Multiplying a Polynomial by a Monomial

Multiplying a polynomial by a monomial is really just applying the Distributive Property of multiplication over addition and subtraction.

Examples: Multiply.

- $(-2x)(3x^2y^4 + 4xy + 2)$
- $3x^2y^2(x^3 + 3xy + 2y + 1)$

Multiplying a Binomial by a Binomial

The multiplication of a binomial by a binomial can be performed by multiplying the each term of one binomial by the other binomial. This is seen by applying the Distributive Property twice.

$$(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$$

We multiply the **F**irst terms, **O**uter terms, **I**nner terms and **L**ast terms. This is abbreviated **FOIL**. FOIL will **ONLY** work with binomials.

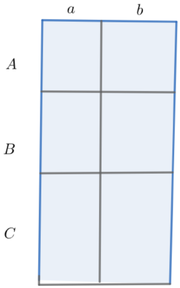
Examples:Multiply.

- $(x + 4)(x - 5)$
- $(x^2 - 6)(x - 2)$
- On Your Own: $(3xy + 2)(4x^2 + y)$

Multiplying a Polynomial by a Polynomial

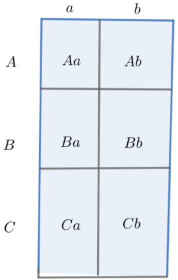
Now we're ready to multiply a polynomial by a polynomial. Remember, FOIL will not work in this case, but we use the Distributive Property. There are various ways to keep track of the terms.

Consider finding the area of the marked figure below:



One way to calculate the area is to multiply the width $(a + b)$ by the length $(A + B + C)$ such that the area is $(a + b)(A + B + C)$.

The other way is to find the area of each little rectangle and then add the results such that the area is $Aa + Ab + Ba + Bb + Ca + Cb$.



Example:Use the distributive property to show that the two areas are the same.

Multiplying a Polynomial by a Polynomial

Examples: Multiply.

- $(y - 3)(y^2 + 4x - 5)$
- On Your Own: $(x^2 - 4x - 6)(x^2 + 4x + 2)$

Multiplying Special Products

Sometimes identifying patterns will make computations 'easier'.

Binomial squares pattern: If a and b are real numbers,

$$\mathbf{(a+b)^2 = a^2 + 2ab + b^2} \qquad \text{and} \qquad \mathbf{(a-b)^2 = a^2 - 2ab + b^2}.$$

Let's use the Distributive Property to prove this.

Examples: Multiply.

- $(2x-4)^2$
- $(3x+6y)^2$

Multiplying Special Products

A **conjugate pair** is two binomials of the form

$$(a-b),(a+b).$$

The pair of binomials each have the same first term and the same last term, but one binomial is a sum and the other is a difference.

Product of conjugates pattern: For any a and b,

$$(a+b)(a-b)=a^2-b^2.$$

The product is called a difference of squares. To multiply conjugates, square the first term, square the last term, write it as a difference of squares.

Multiplying Special Products

Examples: Multiply using the product of conjugates pattern.

- $(3x - 4)(3x + 4)$
- $(5x + 6y)(5x - 6y)$

Multiplying Special Products

Comparing the Special Product Patterns

Binomial Squares	Product of Conjugates
$(a+b)^2 = a^2 + 2ab + b^2$	$(a-b)(a+b) = a^2 - b^2$
$(a-b)^2 = a^2 - 2ab + b^2$	
• Squaring a binomial	• Multiplying conjugates
• Product is a **trinomial**	• Product is a **binomial.**
• Inner and outer terms with FOIL are **the same.**	• Inner and outer terms with FOIL are **opposites.**
• Middle term is **double the product** of the terms	• There is **no** middle term.

Examples:

On Your Own: Choose the appropriate pattern and use it to find the product.

- $(5u-3v)(5u-3v)$
- $(10x-2)(10x+2)$
- $(4+3x)(3-4x)$