Chapter 1.2.2: Evaluating, Adding and Subtracting Polynomials

MAT 1275CO
Dr. Davie

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Positive Integer Exponents

Recall that a positive integer exponent indicates repeated multiplication of the same quantity.

$$(-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8$$

Here, (-2) is the **base** and 3 is the **exponent**.

Polynomials

A **monomial** is an expression formed by multiplying variables and numbers. A **polynomial** is a sum of monomials. We could also say that a **polynomial** is an expression formed by adding (or subtracting) and/or multiplying numbers and variables together.

A **term** of a polynomial is a monomial that is combined with other monomials using addition or subtraction.

- A polynomial with exactly two terms is called a **binomial**.
- A polynomial with exactly three terms is called a **trinomial**.

The coefficient of a term is the number multiplying the product of variables which are then combined via addition.

Polynomial	$y+1$	$4a^2-7ab+2b^2$	$4x^4+x^3+8x^2-9x+1$	0
Monomial	14	$8y^2$	$-9x^3y^5$	$-13a^3b^2c$
Binomial	$a+7ab+7b$	$4x^2-y^2$	y^2-16	$3p^3q-9p^2q$
Trinomial	$x^2-7x+12$	$9m^2+2mn-8n^2$	$6k^4-k^3+8k$	z^4+3z^2-1

Identifying Polynomials

Notice that every monomial, binomial, and trinomial is also a polynomial. They are just special members of the "family" of polynomials and so they have special names. We use the words monomial, binomial, and trinomial when referring to these special polynomials and just call all the rest polynomials.

The **degree of a polynomial** and the degree of its terms are determined by the exponents of the variable.

- The **degree of a term** is the sum of the exponents of its variables.
- A monomial that has no variable, just a constant, is a special case. The **degree of a constant** is 0.
- The **degree of a polynomial** is the highest degree among all its terms.
- A polynomial of degree 1 is called a **linear** expression or polynomial.
- A polynomial of degree 2 is called a **quadratic** expression or polynomial.

Identifying Polynomials

Here are some additional examples.

Monomial	14	$8ab^2$	$-9x^3y^5$	$-13a$
Term	14	$8ab^2$	$-9x^3y^5$	$-13a$
Coefficient of term	14	8	-9	-13
Degree of the monomial	0	3	8	1

Binomial	$h+7$	$7b^2-3b$	x^2y^2-25	$4n^3-8n^2$
Terms	$h, 7$	$7b^2, -3b$	$x^2y^2, -25$	$4n^3, -8n^2$
Coefficients or respective terms	$1, 7$	$7, -3$	$1, -25$	$4, -8$
Degree of respective terms	$1,0$	$2,1$	$4,0$	$3,2$
Degree of the binomial	1	2	4	3

Identifying Polynomials

Trinomial	$x^2 - 12x + 27$	$9a^2 + 6ab + b^2$	$6m^4 - m^3n^2 + 8mn^5$	$z^4 + 3z^2 - 1$
Terms	$x^2, -12x, 27$	$9x^2, 6ab, b^2$	$6m^4, -m^3n^2, 8mn^5$	$z^4, 3z^2, -1$
Coefficient of respective terms	$1, -12, 27$	$9, 6, 1$	$6, -1, 8$	$1, 3, -1$
Degree of respective terms	$2, 1, 0$	$2, 2, 2$	$4, 5, 6$	$4, 2, 0$
Degree of the trinomial	2	2	6	4

Polynomial	$y - 1$	$3y^2 - 2y - 5$	$4x^4 + x^3 + 8x^2 - 9x + 1$
Terms	$y, -1$	$3y^2, -2y, -5$	$4x^4, x^3, 8x^2, -9x, 1$
Coefficient of respective terms	$1, -1$	$3, -2, -5$	$4, 1, 8, 9, 1$
Degree of respective terms	$1, 0$	$2, 1, 0$	$4, 3, 2, 1, 0$
Degree of the polynomial	1	2	4

Identifying Polynomials

Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial. Then, find the degree of each polynomial.

- $8xy^3 + 4xy + 5x + y$
- $-17a^4b^5 + 14a^2b^3 + 1$
- $-x^8y^9z^2$

Evaluating Polynomials and the Order of Operations

When we have an expression that involves exponents, we should evaluate the exponents before multiplying, dividing, adding or subtracting. Also, parentheses should be evaluated before anything else unless you use the distribution property.

Just like we could add and subtract linear expressions (polynomials of degree 1) we can add and subtract polynomials in general by combining like terms.

Examples:

- Find the sum. $(3x^2y - 2x + 2y - 4) + (-4xy^2 + 6x - 2y + 8)$
- Find the difference. $(14y^2 + 6y - 4) - (3y^2 + 8y + 5)$
- Simplify. $(a^3 + a^2b) - (ab^2 + b^3) + (a^2b + ab^2)$

Applications

Examples: The polynomial $-16t^2 + 250$ gives the height of a ball t seconds after it is dropped from a 250-foot tall building. Find the height after $t = 2$ seconds. In this example the variable t is a place holder for any number you may be interested in rather than an 'unknown' number.