Chapter 1.1.4: Integer Exponents

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Integer Exponents

Exponential Notation

The positive integer exponent indicates repeated multiplication of the same quantity. For example, in the expression a^m the positive integer exponent m tells us how many times we use the base a as a factor.

$$a^3 = a \cdot a \cdot a \qquad\qquad a^6 = a \cdot a \cdot a \cdot a \cdot a \cdot a$$

The expression a^m has base a and exponent m. This is read a to the mth (**power**), or a to the power of m.

In the expression a^m with positive integer m and $a \neq 0$, the **exponent** m tells us how many times we use the **base** a as a factor.

Integer Exponents

Examples:

- Evaluate 2^4.
- Evaluate -5^2.
- Evaluate $(-5)^2$.
- Write using exponents: $-3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.
- Identify the base and the exponent of -4^2.

Properties of Exponents

Exponents have special properties.

Product property for positive integer exponents: If a, m and n are real numbers, then

$$a^m a^n = a^{m+n}.$$

To multiply with like bases, add the exponents.

Examples:Simplify each expression.

- $2^8 2^7$
- $11^2 11^4 11^{3 \cdot 6}$

Properties of Exponents

Exponents have special properties.

Quotient property for positive integer exponents: If $a \neq 0, m, n$ are real numbers, then

$$\frac{a^m}{a^n} = a^{m-n}.$$

To divide with like bases, subtract the exponents.

Examples:Simplify each expression.

- $\frac{2^8}{2^5}$
- $\frac{11^4}{11^3}$

Properties of Exponents

Power property for integer exponents: If a, m and n are real numbers, then

$$(a^m)^n = a^{mn}.$$

To raise a power to a power, multiply the exponents.

Examples:Simplify each expression.

- $(2^5)^3 2^7$
- $(12^4)^5 12$

Properties of Exponents

Product to a Power Property for Integer Exponents: If a, b and m are real numbers, then

$$(ab)^m = a^m b^m.$$

To raise a product to a power, raise each factor to that power.

Examples:Simplify each expression using the Product to a Power Property.

- $(-2 \cdot 4)^3$
- $(5 \cdot 9^3)^2$

Properties of Exponents

Quotient to a Power Property for Integer Exponents: If a, $b \neq 0$ and m are real numbers, then

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}.$$

To raise a quotient to a power, raise the numerator and the denominator to that power.

Examples:Rewrite using the Quotient to a Power Property.

- $(\frac{3}{4})^3$

Properties of Exponents

Zero Exponent Property: For any non-zero real number a,

$$a^0 = 1.$$

Negative Exponent Property: For any real numbers a and m,

$$a^{-m} = \frac{1}{a^m} \text{ or, equivalently, } \frac{1}{a^{-m}} = a^m.$$

Examples:Simplify each expression.

- $(-6)^0$
- 2^{-5}
- $\frac{1}{3^{-4}}$
- $-(6^2 \cdot 5 \cdot 4)^0$

Properties of Negative Exponents

Quotient to a Negative Exponent Property: If a, b and n are real numbers, $a \neq 0$ and $b \neq 0$, then

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}.$$

Examples:Simplify each expression. Write your answer using positive exponents.

- $\left(\frac{3}{7}\right)^{-2}$

Properties of Exponents

Definition	**Description**
Definition of Zero Exponent	$a^0 = 1, a \neq 0$
Definition of Negative Exponents	$a^{-n} = \frac{1}{a^n}$, or equivalently, $\frac{1}{a^{-n}} = a^n$
Property	**Description**
Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power	$(ab)^n = a^n b^n$
Quotient Property	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$
Quotient to a Negative Exponent	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Properties of Exponents

Examples:Simplify each expression. Write your answer using positive exponents.

- $\left(\frac{3 \cdot 5^{-3}}{(-2)^{-5}}\right)^{-3}$
- $(\frac{2}{5})^{-2} + 7^0 - (\frac{2}{3})^2$