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**DIGITAL
MORPHOGENESIS**

In contemporary architectural design, digital media is increasingly being used not as a representational tool for visualization but as a generative tool for the derivation of form and its transformation – *the digital morphogenesis*. In a radical departure from centuries-old traditions and norms of architectural design, digitally-generated forms are not designed or drawn as the conventional understanding of these terms would have it, but they are calculated by the chosen generative computational method. Instead of modeling an external form, designers articulate an internal generative logic, which then produces, in an automatic fashion, a range of possibilities from which the designer could choose an appropriate formal proposition for further development.

The predictable relationships between design and representations are abandoned in favor of computationally-generated complexities. Models of design capable of consistent, continual and dynamic transformation are replacing the static norms of conventional processes. Complex curvilinear geometries are produced with the same ease as Euclidean geometries of planar shapes and cylindrical, spherical or conical forms. The plan no longer “generates” the design; sections attain a purely analytical role. Grids, repetitions and symmetries lose their past *raison d’être*, as infinite variability becomes as feasible as modularity, and as mass-customization presents alternatives to mass-production.

The digital generative processes are opening up new territories for conceptual, formal and tectonic exploration, articulating an architectural morphology focused on the emergent and adaptive properties of form. The emphasis shifts from the “making of form” to the “finding of form,” which various digitally-based generative techniques seem to bring about intentionally. In the realm of form, the stable is replaced by the variable, singularity by multiplicity.

TOPOLOGY

Computational, digital architectures are defined by computationally-based processes of form origination and transformations, i.e. the processes of digital morphogenesis, where the plural (“architectures”) emphasizes multiplicities inherent in the logics of the underlying computational concepts, such as topological geometries, isomorphic polysurfaces (“blobs”), motion kinematics and dynamics, keyshape animation (metamorphosis), parametric design, genetic

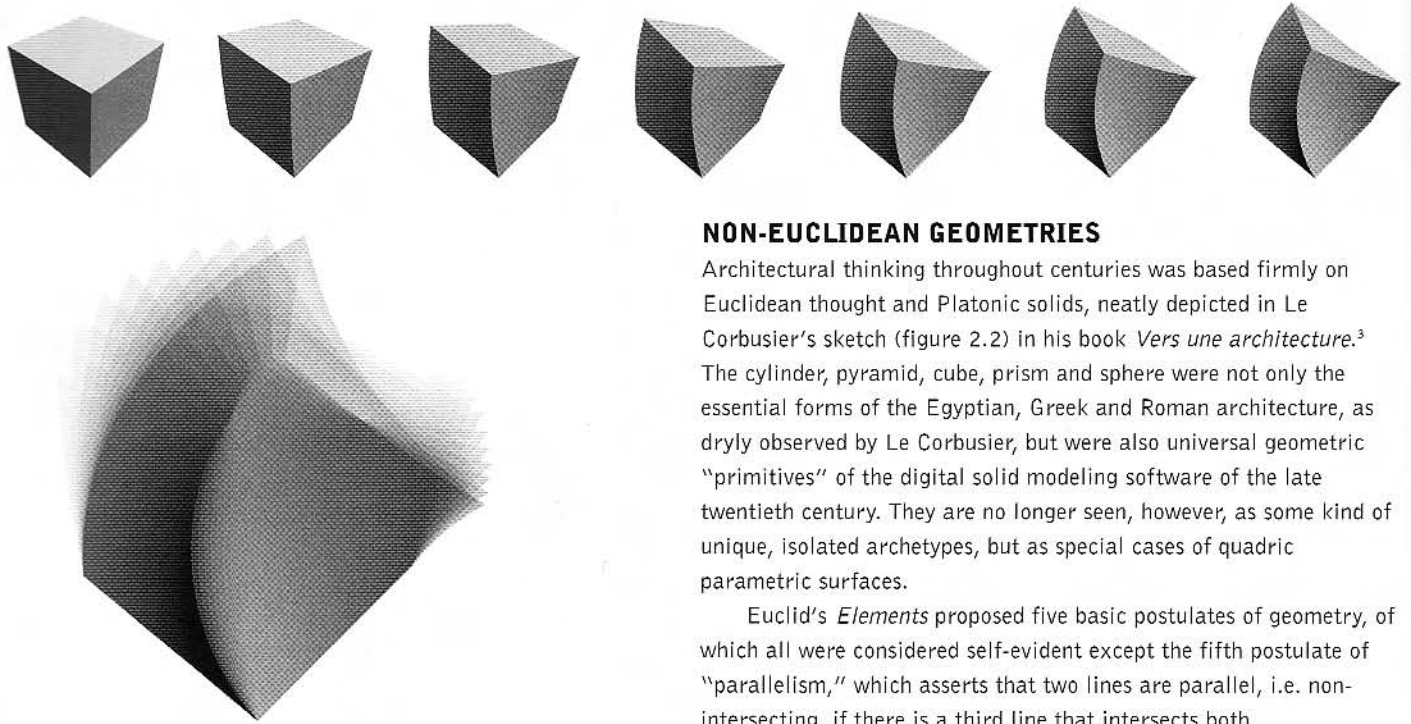
algorithms (evolutionary architectures), performance, etc., which are discussed in more detail in the following sections.

The notion of topology has particular potentiality in architecture, as emphasis shifts away from particular forms of expression to relations that exist between and within an existing site and the proposed program. These interdependences then become the structuring, organizing principle for the generation and transformation of form.

According to its mathematical definition, topology is a study of intrinsic, qualitative properties of geometric forms that are not normally affected by changes in size or shape, i.e. which remain invariant through continuous one-to-one transformations or elastic deformations, such as stretching or twisting. A circle and an ellipse, for example, or a square and a rectangle, can be considered to be topologically equivalent, as both circle and square could be deformed by stretching them into an ellipsoid or rectangle, respectively. A square and a rectangle have the same number of edges and the same number of vertices, and are, therefore, topologically identical, or homeomorphic. This quality of homeomorphism is particularly interesting, as focus is on the relational structure of an object and not on its geometry – the same topological structure could be geometrically manifested in an infinite number of forms (figure 2.1). Topological transformations, first and foremost, affect the relational structure, and, thus, the resulting form(s). For example, a rectangle could be transformed into a triangle with a single topological operation of deleting one of its vertices.

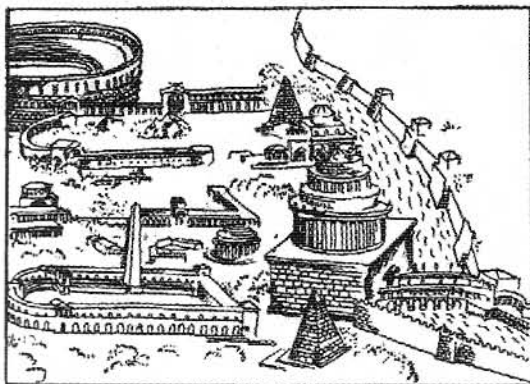
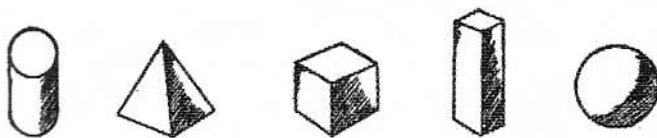
Because of their intrinsic property of one-sidedness, topological structures such as the Möbius strip¹ (figure 1.12 in Chapter 1) and the Klein bottle² (figure 1.13 in Chapter 1), have a potential for an architecture in which the boundaries between what is interior and what is exterior are blurred, an architecture that avoids the normative distinctions of “inside” and “outside.” While the conceptual possibilities of these topological geometries are intriguing, their inherent, conceptual qualities are often difficult to truly manifest tectonically, as *Möbius House* (1995) by Ben Van Berkel and Caroline Bos shows to some extent. The transparent and solid boundaries of the shelter, which a house must provide, often work against the seamless continuities and erasure of inside/outside dichotomy imbued within the Möbius strip. What makes topology particularly appealing are not the complex forms, such as the Möbius strip, but the primacy over form of the structures of relations, interconnections or inherent qualities which exist internally and externally within the context of an architectural project.

graphic
ally
figures.



Because topological structures are often represented by complex, curvilinear forms, topology is popularly – and wrongly – considered synonymous with curved surfaces. Another common misnomer is to refer to topologically produced geometries as “non-Euclidean.” As soon as a topological structure is given a geometric, architectonic form, the operative realm is firmly Euclidean. As the following section demonstrates, both Euclidean and non-Euclidean geometries are part of the same geometric universe, in which the Euclidean geometry is simply one special case, albeit one that has been firmly established in architectural thought and practice over the last few centuries.

er:
c composition
architecture.



NON-EUCLIDEAN GEOMETRIES

Architectural thinking throughout centuries was based firmly on Euclidean thought and Platonic solids, neatly depicted in Le Corbusier’s sketch (figure 2.2) in his book *Vers une architecture*.³ The cylinder, pyramid, cube, prism and sphere were not only the essential forms of the Egyptian, Greek and Roman architecture, as dryly observed by Le Corbusier, but were also universal geometric “primitives” of the digital solid modeling software of the late twentieth century. They are no longer seen, however, as some kind of unique, isolated archetypes, but as special cases of quadric parametric surfaces.

Euclid’s *Elements* proposed five basic postulates of geometry, of which all were considered self-evident except the fifth postulate of “parallelism,” which asserts that two lines are parallel, i.e. non-intersecting, if there is a third line that intersects both perpendicularly. The consequence of this postulate in Euclidean geometry is that through every point there is one and only one line parallel to any other line.

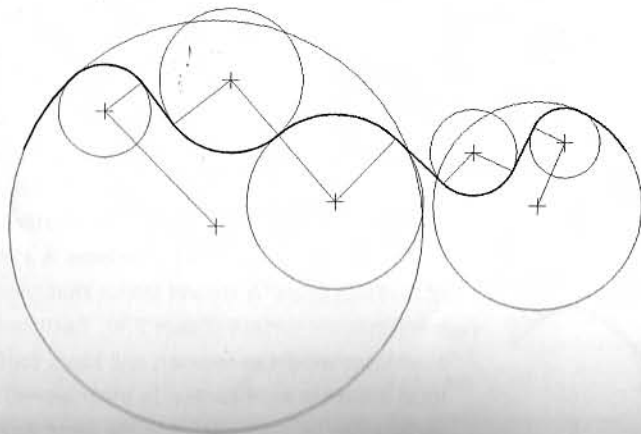
The first four postulates, as articulated by Euclid, are considered postulates of absolute geometry. It was this fifth postulate that opened the realm of non-Euclidean geometries. Though many had questioned Euclid’s fifth postulate, it was Carl Friedrich Gauss and the mathematicians after him who have finally managed to successfully demonstrate the existence of non-Euclidean geometries. The publication of Eugenio Beltrami’s seminal *Essay on an Interpretation of Non-Euclidean Geometry* in 1868 showed beyond doubt that curved lines could appear straight, that spherical geometry could seem planar, and that curved space could appear Euclidean, i.e. flat, thus turning the worlds of physics and astronomy upside down.⁴ Albert Einstein’s “Theory of Relativity,” based on non-Euclidean geometry, powerfully showed how Newtonian physics, based upon Euclidean geometry, failed to consider the essential curvature of space.

The work of Gauss, Lobachevsky, Riemann, von Helmholtz, and other mathematicians and physicists later on, showed that space is not only curved but also multi-dimensional. By showing that geometries could be based on non-Euclidean relationships (such as parallelism, for example), they opened up other spatial possibilities disconnected from empirical intuition.⁵ In Riemannian geometry, which is also known as “spherical” geometry, the “plane” is situated on the surface of a sphere, and the “line” is a circle that has the same radius as the sphere. For every two points, there is one and only one circle that connects them; as a consequence of this definition and the underlying spherical geometry, no parallel “lines” exist in

Riemannian geometry, and every infinite "line," i.e. circle, intersects every other infinite "line." Also, the distance between two points is always a curved distance, i.e. not a "flat" distance. In Poincaré geometry, for example, "lines" are hyperbolas on a Cartesian plane; there is an infinite number of "lines" through a chosen point that are parallel to another "line."

Each of these non-Euclidean geometries has a particular application. Riemannian geometry is used in navigation, and Poincaré geometry is used in ballistics and for the representation of electromagnetic forces. What makes these and other non-Euclidean geometries interesting from an architectural point of view is the possibility of mapping objects between them, thus providing for a radically different conceptualization of space. Some modeling software, for example, provides for limited transformations of the Cartesian modeling space, which can approximate spatial characteristics of some of the non-Euclidean geometries.

Another interesting concept, which Bernhard Riemann introduced, is the concept of curvature of space and the spaces of positive and negative curvature. In this definition of space, Euclidean "flat," planar space occupies the median position, having zero curvature. Euclidean geometry is then just a special kind of geometry, a special point on the infinite scale of bending, or folding, that produces "flatness" as a manifestation of an equilibrium that is established among various influences producing the curving of space in the first place. In other words, in the Riemannian conception of space, the "boxes" and "blobs" are simply instances on a sliding scale of formal complexity – a box could be turned into a blob and vice versa by simply varying the parameters of space within which they are defined.



44
A composite curve constructed from tangent circular arcs and straight line segments.

As architectural conceptions of space move from the three dimensions of the Cartesian space to fourth-dimensional continuum of interactions between space and time, other dimensions and other conceptions of space begin to open up intriguing possibilities, which may or may not offer new potentialities for architectural thought. An architecture of warped multidimensional space would move beyond the mere manipulation of shapes and forms into the realm of events, influences and relationships of multiple dimensions.

NURBS

In pre-digital architecture, whose formal potentiality was, in large part, a direct extension of the limits of Euclidean geometry (lines, circles, quadrilaterals, etc.), the description and, consequently, the construction of compound, complex curves was accomplished through an approximation by concatenating tangent circular arcs and straight line segments (figure 2.3), which could be delineated with ease on paper and on the building site.

The introduction of digital modeling software into architectural design provided a departure from the Euclidean geometry of discrete volumes represented in Cartesian space and made possible the present use of "topological," "rubber-sheet" geometry of continuous curves and surfaces that feature prominently in contemporary architecture. The highly curvilinear surfaces in the architecture of the digital avant-garde are described mathematically as NURBS, which is an acronym that stands for Non-Uniform Rational B-Splines. What makes NURBS curves and surfaces particularly appealing is their ability to easily control their shape by interactively manipulating the *control points*, *weights* and *knots*. NURBS make the heterogeneous, yet coherent, forms of the digital architectures computationally possible and their construction attainable by means of computer numerically controlled (CNC) machinery.

But why NURBS? The main reason for their widespread adoption is the ability of NURBS to construct a broad range of geometric forms, from straight lines and Platonic solids to highly complex, sculpted surfaces. From a computational point of view, NURBS provide for an efficient data representation of geometric forms, using a minimum amount of data and relatively few steps for shape computation, which is why most of today's digital modeling programs rely on NURBS as a computational method for constructing complex surface models and, in some modelers, even solid models.

NURBS are a digital equivalent of the drafting splines used to draw the complex curves in the cross-sections of ship hulls and airplane fuselages. Those splines were flexible strips made of plastic, wood or metal that would be bent to achieve a desired smooth curve,

curve can be changed by interactively manipulating the control points, weights and knots.

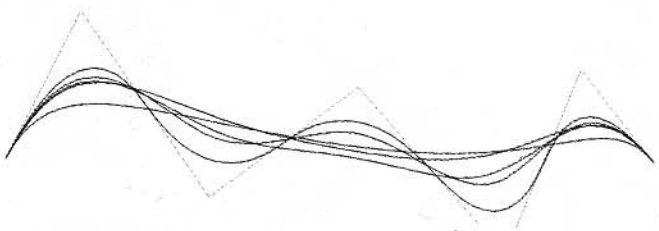


with weights attached to them in order to maintain the given shape. The term spline (the "S" in NURBS) actually has its origin in shipbuilding, where it was used to refer to a piece of steamed wood shaped into a desired smooth curve and kept in shape with clamps and pegs. Mathematicians borrowed the term in a direct analogy to describe families of complex curves.

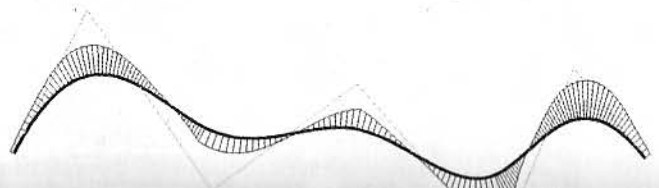
The shape of a NURBS curve can be changed by manipulating its *control points* and associated *weights* and *knots* (figure 2.4), as well as the *degree* of the curve itself (figure 2.5). The NURBS curves are shaped primarily by changing the location of control points, which do not have to lie on the curve itself, except for the endpoints. Each control point has an associated weight, which determines the extent of its influence over the curve, in a direct analogy to drafting splines. Increasing the weight of a control point pulls the corresponding curve or surface toward that control point and vice versa.

Each control point has an associated polynomial equation, commonly referred to as a basis function (the "B" in NURBS, and in B-splines in general). A rational B-spline (the "R" in NURBS) is defined mathematically as the ratio of two polynomial equations, i.e. two basis functions. Each basis function affects only the curve section in the vicinity of the associated control point, and these sections are delimited by knots. A non-uniform rational B-spline is one in which the influence of a control point (i.e. the associated basis function) on a curvature can be varied by changing the location of the knots along the control segment that links two control points; in other words, a non-uniform rational B-spline is one with unequal knot spacing.

2.5.
Varying the degree of a NURBS curve will produce different shapes.



2.6.
Curvature graph for a cubic Bezier spline.



Another important parameter that can affect the shape of a NURBS curve is the degree, i.e. the highest exponent within the polynomial equations associated with control points. The lower the polynomial degree, the closer the curve is placed towards the control points. Thus, the second degree (quadratic) basis functions would pull the curve closer to control points than the third degree (cubic) ones (figure 2.5). The first degree (linear) functions produce a "curve" with straight line segments.

Other spline curves, as subcategories of NURBS, are typically available in modeling software. *B-splines* are actually NURBS with equally weighted control points (thus, weights are not displayed). *Bézier* curves, named after Pierre Bézier, the French automotive engineer who invented them, are B-splines with equal knot spacings (thus, knots are not shown). Cubic curves are actually third-degree continuous Bézier curves, and quadratic curves are second-degree continuous Bézier curves. In this pseudo-taxonomy of spline curves, at each level an additional set of controls over curvature is lost: weights in the case of B-splines, and both weights and knots in the case of Bézier curves.⁶

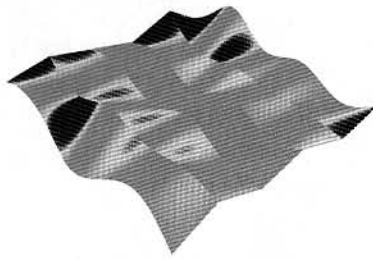
An important property of curves made by splines is that their curvature, i.e. the curve radius, changes continually along their length, in sharp contrast to curves made of tangent circular arcs, which, despite their smooth appearance, have discrete points at which the curvature changes abruptly. There are different levels of curvature continuity (figure 2.6): a curve with an angle or a cusp is said to have C0 continuity;⁷ a curve without cusps but with changing curvature has C1 continuity;⁸ a curve with constant curvature is C2 continuous⁹ – higher levels of continuity are possible, but for most practical purposes, these three levels are sufficient. Besides fluid dynamics, the curvature continuity also has important aesthetic and manufacturing implications, which is why most modeling programs provide tools for the continuity analysis (figures 2.7 and 2.8).

The location of control points in a NURBS curve can affect its continuity locally, meaning that different segments can have different levels of continuity. For instance, two coincident control points in a NURBS curve would pronounce the curvature; three coincident control points would produce an angular cusp. This potentiality of NURBS curves of having varying continuity is referred to as *multiplicity*.

The definition of NURBS surfaces is a straightforward extension of NURBS curves. A control lattice that connects control points surrounds the surface (figure 2.9). Each control point has an associated weight parameter, and knots control the distribution of the

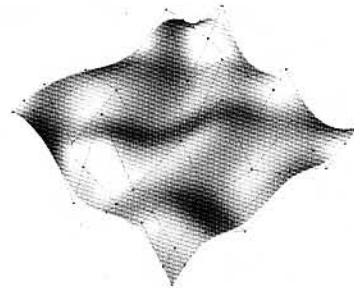
2.7.

Curvature continuity:
the *zebra* analysis.



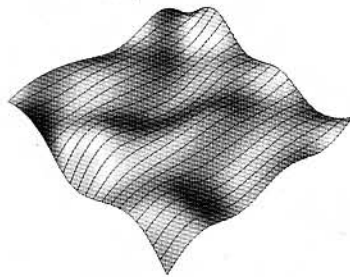
2.8.

Curvature continuity:
the *Gaussian* analysis.



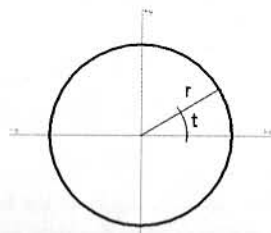
2.9.

The control lattice for
a NURBS surface.



2.10.

Isoparametric contours
in the "U" direction of
a NURBS surface.



$$\begin{aligned}x &= r \cdot \cos t \\y &= r \cdot \sin t \\0 &\leq t \leq \pi/2\end{aligned}$$

2.11.

Parametric definition
of a circle.

Another property of NURBS objects, which is of particular importance from a conceptual point of view, is that they are defined within a "local" *parametric* space, situated in the three-dimensional Cartesian geometric space within which the objects are represented.¹⁰ That parametric space is one-dimensional for NURBS curves, even though the curves exist in a three-dimensional geometric space. That one-dimensionality of curves is defined at a topological level by a single parameter commonly referred to as "U." Surfaces have two dimensions in the parametric space, often referred to as "U" and "V" in order to distinguish them from X, Y and Z of the Cartesian three-dimensional geometric realm. Isoparametric curves ("isoparms") are used to aid in the visualizing of NURBS surfaces through contouring in the "U" and "V" direction (figure 2.10). These curves have a constant U or V parameter in the parametric NURBS math, and are similar to topographic contour lines that are used to represent constant elevations in landscape.

The parametric description of forms (*parametrics*) provides a particularly versatile way to represent complex curves and surfaces. Sets of equations are used to express certain quantities as explicit functions of a number of variables, i.e. parameters, which can be independent or dependent. For instance, one set of parametric equations for a circle in two-dimensional Cartesian coordinate space could be given as $x = r \cdot \cos t$ and $y = r \cdot \sin t$, whereby the parameter t is the inscribed angle whose value can range from 0 to 2π (figure 2.11). Parametric representations are generally non-unique, i.e. the same quantities can be expressed by a number of different parameterization strategies (for example, the equation $r^2 = x^2 + y^2$ is another way to describe the geometry of the circle).

PARAMETRICS

Parametrics can provide for a powerful conception of architectural form by describing a range of possibilities, replacing in the process stable with variable, singularity with multiplicity. Using parametrics, designers could create an infinite number of similar objects, geometric manifestations of a previously articulated schema of variable dimensional, relational or operative dependencies. When those variables are assigned specific values, particular instances are created from a potentially infinite range of possibilities.

In parametric design, it is the parameters of a particular design that are declared, not its shape. By assigning different values to the parameters, different objects or configurations can be created. Equations can be used to describe the relationships between objects, thus defining an *associative* geometry – the "constituent geometry that is mutually linked."¹¹ That way, interdependencies between

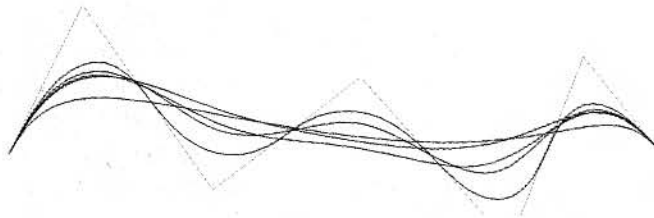
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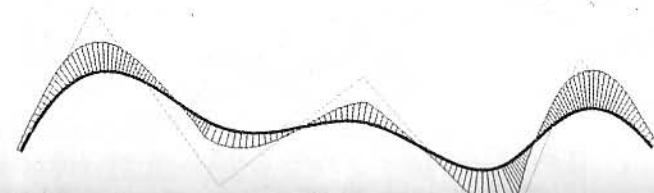
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2.5.
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Curvature graph for a cubic Bezier spline.



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The definition of NURBS surfaces is a straightforward extension of NURBS curves. A control lattice that connects control points surrounds the surface (figure 2.9). Each control point has an associated weight parameter, and knots control the distribution of the local influence as in curves. In other words, the shape of a NURBS



Marcos

objects can be established, and objects' behavior under transformations defined. As observed by Burry, "the ability to define, determine and reconfigure geometrical relationships is of particular value."¹²

Parametric design often entails a procedural, algorithmic description of geometry. In his "algorithmic spectaculars" (figures 2.12a–d), i.e. algorithmic explorations of "tectonic production" using *Mathematica* software, Marcos Novak constructs "mathematical models and generative procedures that are constrained by numerous variables initially unrelated to any pragmatic concerns... Each variable or process is a 'slot' into which an external influence can be mapped, either statically or dynamically."¹³ In his explorations, Novak is "concerned less with the manipulation of objects and more with the manipulation of relations, fields, higher dimensions, and eventually the curvature of space itself."¹⁴ The implication is that the parametric design does not necessarily predicate stable forms. As demonstrated by Burry, one can devise a *paramorph* – an unstable spatial and topological description of form with stable characteristics (figure 2.13).

The *International Terminal* at Waterloo Station in London (1993, figure 2.14), by Nicholas Grimshaw and Partners, offers a clear demonstration of conceptual and developmental benefits afforded by the parametric approach to design. The building is essentially a 400 m long glass-clad train shed, with a "tapering" span that gradually shrinks from 50 m to 35 m. Its narrow,

sinuous plan is determined by the track layout and the difficult geometry of the site, which is the main source of the project's complexity and which gives such potency and significance to Grimshaw's design, especially its spectacular roof structure.

The roof structure consists of a series of 36 dimensionally different but identically configured three-pin bowstring arches (figure 2.15). Because of the asymmetrical geometry of the platforms, the arches rise steeply on one side with a shallower incline over the platforms on the other side. Each arch is different as the width of the roof changes along the tracks.

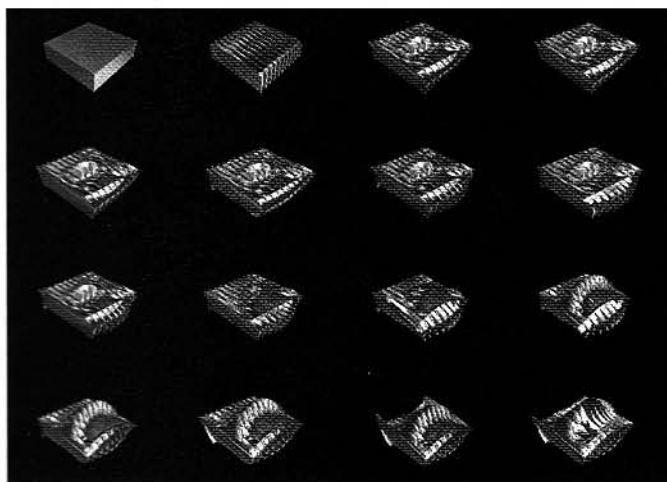
Instead of modeling each arch separately, a generic parametric model was created based on the underlying design rules in which the size of the span and the curvature of individual arches were related (figures 2.16a–b). By assigning different values to the span parameter, 36 dimensionally different, yet topologically identical, arches were computed and inserted in the overall geometric model.

The parametric model could be extended from the structural description of arches to the elements that connect them, the corresponding cladding elements, i.e. to the entire building form. Thus, a highly complex hierarchy of interdependences could be parametrically modeled, allowing iterative refinement, i.e. the dimensional fine-tuning of the project in all stages of its development, from conceptual design to construction.

As shown by this project, parametrics are particularly useful for modeling the geometry of complex building forms. Their successful application requires careful articulation of a clear strategy of tectonic resolution, such that a sufficiently clear description of interdependences can be achieved; in other words, a well-defined design strategy is essential for the effective application of parametrics.

Parametric approach to design, if consistently applied from its conceptual phase to its materialization, profoundly changes the entire nature and the established hierarchies of the building industry, as well as the role of the architect in the processes of building. For the first time in history, architects are designing not the specific shape of the building but a set of principles encoded as a sequence of parametric equations by which specific instances of the design can be generated and varied in time as needed. Parametric design calls for the rejection of fixed solutions and for an exploration of infinitely variable potentialities.

by Mark



configured three-pin
bowstring arches.

Partners.

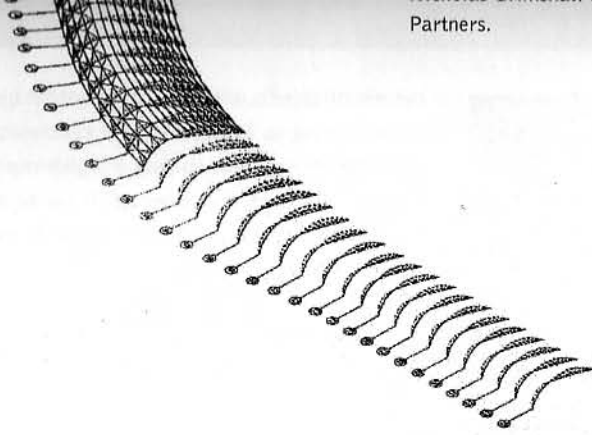
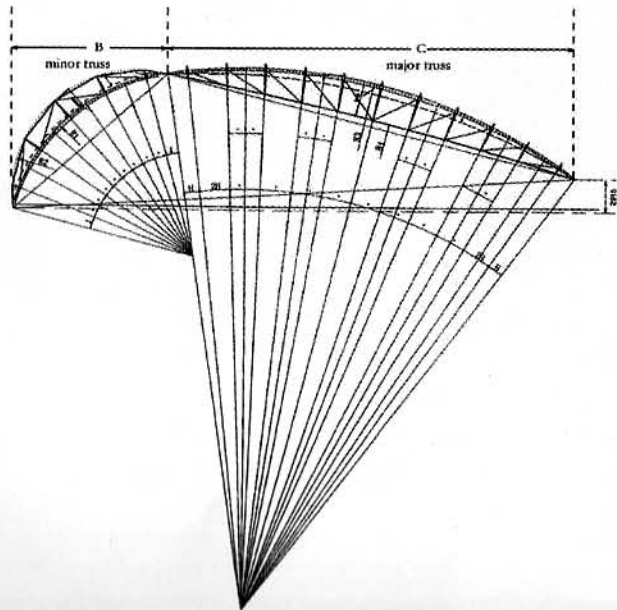
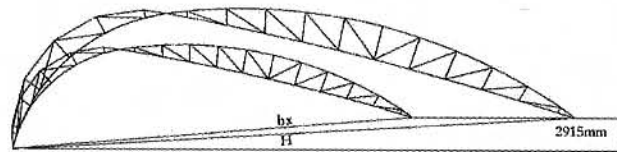


Fig. 16a-b.
Parametric definition of
the scaling factor for
the truss geometry:
 $k_x = (2915^2 + (B + C)^2)^{0.5}$,
where B is the minor
truss span and C is the
major truss span.



DYNAMICS AND FIELDS OF FORCES

As Greg Lynn observed in *Animate Form*,¹⁵ "it is important for any parameter-based design that there be both the unfolding of an internal system and the infolding of contextual information fields." Architectural form, in other words, is not only a manifestation of its internal, parameter-driven relational logics, but it also has to engage and respond to dynamic, often variable influences from its environmental and socio-economic context. Architectural form, instead of being conceived as a stationary, inert construct, is conceptually a highly plastic, mutable entity that evolves dynamically through its transformative interactions with external, gradient forces. According to Lynn, in place of a neutral abstract space, "the context of design becomes an active abstract space that directs from within a current of forces that can be stored as information in the shape of the form."¹⁶

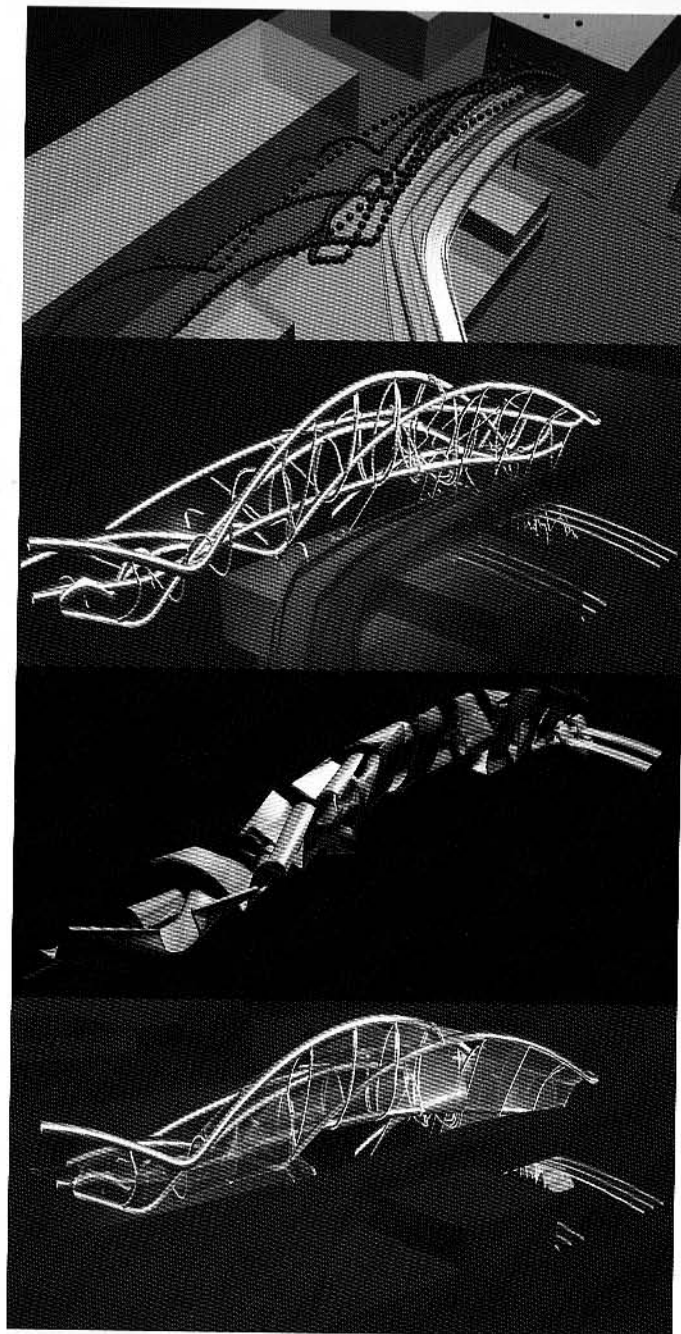
Greg Lynn was one of the first architects to utilize animation software not as a medium of representation, but of form generation. He asserts that the prevalent "cinematic" model of motion in architecture eliminates force and motion from the articulation of form and reintroduces them later, after the fact of design, through concepts and techniques of optical procession. In contrast, as defined by Lynn, "animate design is defined by the co-presence of motion and force at the moment of formal conception."¹⁷ Force, as an initial condition, produces as its effects both motion and particular inflections of form. According to Lynn, "while motion implies movement and action, animation implies evolution of a form and its shaping forces."¹⁸

In his seminal projects, showcased in *Animate Form*, Lynn utilizes an entire repertoire of motion-based modeling techniques, such as keyframe animation, forward and inverse kinematics, dynamics (force fields), and particle emission. Kinematics, in its true mechanical meaning, is used to study the motion of an object or a hierarchical

used in the *House Prototype in Long Island* project, architect Greg Lynn.

2.18a-d.

The use of particle emission in the *Port Authority Bus Terminal* in New York competition project, architect Greg Lynn.



2.19.

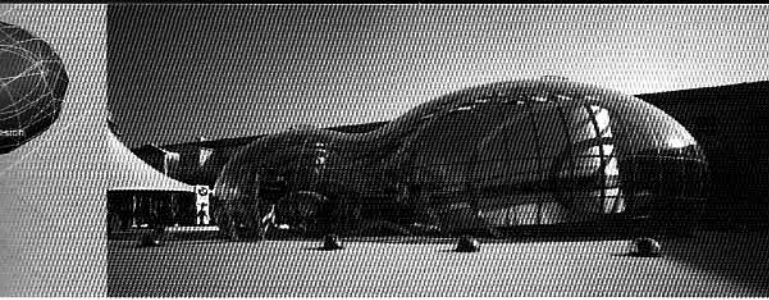
Form can be generated by subjecting the basic structures to force fields extrapolated from the context of the project.

system of objects without consideration given to its mass or the forces acting on it. Hierarchical constructs, such as "skeletons" made of "bones" and "joints," which can have various associated constraints, allow designers to create an infrastructure of relations that determine the complex behavior of the model under transformations, which, for example, can result from the influence of external forces. A "global skin" assigned to such "skeletal" hierarchical organizations makes the deformations formally manifestable. As motion or external influences are applied, transformations are propagated down the hierarchy in forward kinematics, and upwards in inverse kinematics. In some of Lynn's projects, such as the *House Prototype in Long Island* (figures 2.17a-c), skeletons with a global envelope are deformed using inverse kinematics under the influence of various site-induced forces.

In contrast to kinematics, the dynamic simulation takes into consideration the effects of forces on the motion of an object or a system of objects, especially of forces that do not originate within the system itself. Physical properties of objects, such as mass (density), elasticity, static and kinetic friction (or roughness), are defined. Forces of gravity, wind, or vortex are applied, collision detection and obstacles (deflectors) are specified, and dynamic simulation computed. Gradient field influences are applied as direct abstract analogies for environmental influences, such as wind and sun, and contextual phenomena, such as pedestrian and vehicular movements, urban vistas, configurations, patterns and intensities of use, etc. Greg Lynn's design of a protective roof and a lighting scheme for the bus terminal in New York (Figures 2.18a-d) offers a very effective example of using particle systems to visualize the gradient fields of "attraction" present on the site, created by the forces associated with the movement and flow of pedestrians, cars and buses across the site.

The incorporation of movement into what was, by definition, static and unmovable is nothing new – it was one of the ideals of modern architecture. However, the architecture that was described by modernists as embodying movement simply promoted movement through its interior and exterior, becoming, as observed by Ignasi de Sola Morales, "above all a space for mobility, a container in which movement was prefigured."¹⁹





The architecture of motion, therefore, is not the same as the architecture of movement.²⁰ It prioritizes form over space by introducing the motion and force at the moment of formal conception.²¹ It is the dynamics of forces, or, more precisely, force fields, as an initial condition that produces the motion and the particular transformations of form, i.e., the digital morphogenesis (figure 2.19). The form and its changes become products of the dynamic action of forces, a proposition adopted by Lynn directly from D'Arcy Thompson's *On Growth and Form*, published in 1917, in which Thompson argues that the form in nature and the changes of form are due to the "action of force."²² One of Lynn's principal arguments is that "traditionally, in architecture, the abstract space of design is conceived as an ideal neutral space of Cartesian coordinates," but that in other design fields, "design space is conceived as an environment of force and motion rather than as a neutral vacuum."²³ According to Lynn, "while physical form can be defined in terms of static coordinates, the virtual force of the environment in which it is designed contributes to its shape,"²⁴ thus making the forces present in the given context fundamental to the form making in architecture. Lynn attributes to this position the significance of a paradigm shift "from a passive space of static coordinates to an active space of interactions," which he describes as "a move from autonomous purity to



contextual specificity."²⁵ Instrumental to this conceptual shift is the use of digital media, such as animation and special-effects software, which Lynn uses as tools for design rather than as devices for rendering, visualization and imaging.

Instead of subjecting generic formal constructs to the influences of force fields, designers could directly visualize the shape of the force fields using *isomorphic polysurfaces*, which represent yet another point of departure from Platonic solids and Cartesian space. Blobs or metaballs, as isomorphic polysurfaces are sometimes called, are amorphous objects constructed as composite assemblages of mutually-inflecting parametric objects with internal forces of mass and attraction. They exercise fields or regions of influence (figure 2.20), which could be additive (positive) or subtractive (negative). The geometry is constructed by computing a surface at which the composite field has the same intensity – hence the name isomorphic polysurfaces.

Isomorphic polysurfaces open up yet another formal universe where forms may undergo variations, giving rise to new possibilities. Objects interact with each other instead of just occupying space; they become connected through a system of interactions where the whole is always open to variations as new blobs (fields of influence) are added or new relations made, creating new possibilities. The surface boundary of the whole (the isomorphic polysurface) shifts or moves as fields of influence vary in their location and intensity (figures 2.21a–b). In that way, objects begin to operate in a temporally-conditioned dynamic, rather than a static geography.

DATASCAPES

With his pioneering work on using motion dynamics to generate architectural form, Lynn has convincingly demonstrated what Nicholas Negroponte had only hinted at in his seminal work from some 30 years ago, *The Architecture Machine*, and which is also acknowledged in Lynn's writing:

"Physical form, according to D'Arcy Thompson, is the resolution at one instant of time of many forces that are governed by rates of change. In the urban context the complexity of these forces often surpasses human comprehension. A machine, meanwhile, could procreate forms that respond to many hereto un-manageable dynamics. Such a colleague would not be an omen of professional retirement but rather a tickler of the architect's imagination, presenting alternatives of form possibly not visualized or not visualizable by the human designer."²⁶

Buildings and projects in general are conceived within a complex web of planning and building regulations (which are by no means fixed constructs), various technical constraints, environmental conditions, such as sun, wind, precipitation, etc., and are meant to operate in a highly dynamic socio-economic and political context, which has its own "force fields" such as, for instance, numerous interest groups. Some of these influences could be quantified and their changes modeled in order to simulate past, and predict present and future, impact.



2.23.
Ústra Office Building
(1999), Hanover,
Germany, architect
Frank Gehry.

The design approach of the Dutch firm MVRDV acknowledges explicitly the existence of these "gravity fields" and their principal role in the shaping of the built environment (figure 2.22). In order to harvest the informational potential of the complexities inherent in various forces and the complex web of their interactions, MVRDV came up with the concept of *datascares*,²⁷ which are visual representations of quantifiable forces that could influence or impact the conception and development of design projects. These informational landscapes become essential in understanding how these intangible influences manifest themselves in the built environment and how societal, economic, political and cultural fluxes and shifts influence contemporary architecture.

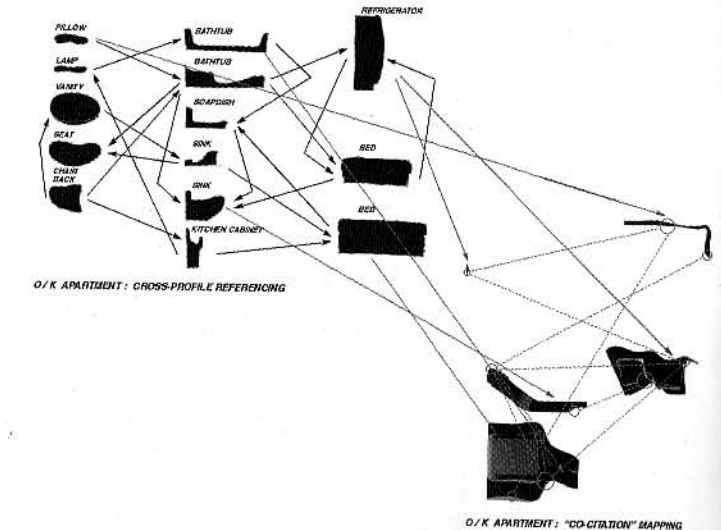
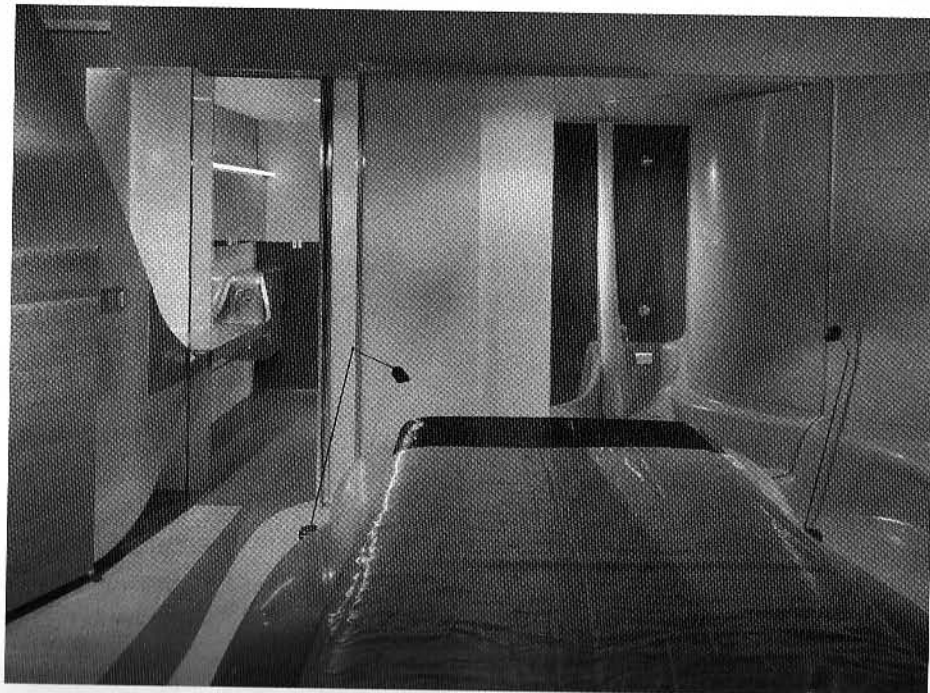
In MVRDV's approach, for each influence a separate *datascape* is constructed. Various *datascares*, relevant for the selected context, are then superposed, creating a complex spatial envelope, with often contradictory, paradoxical conditions, which embodies within its limits the inherent possibilities for the genesis of an architectural project. The challenge, of course, is how to avoid a literal transcription of the diagrams of contextual flows and forces into an architectural form, as the superposition of *datascares*, static or dynamic, often generates spatial and temporal constructs with apparent architectonic qualities.

METAMORPHOSIS

Digital modeling software offers a rich repertoire of transformations a designer could use to further explore formal potentialities of an already conceived geometry. Simple, topologically invariant transformations, such as twisting and bending, are particularly effective means for creating alternative morphologies. For instance, Gehry's *Ústra Office Building* in Hanover, Germany (1999), has a simple prismatic form, which twists in the direction of the nearby open park area (figure 2.23).

By adding a fourth, temporal dimension to the deformation processes, animation software adds a possibility to literally express the space and form of an object's metamorphosis. In *keyshape* (keyframe) animation, different states of an object (i.e. *keyshapes* or *keyframes*) are located at discrete points in time, and the software then computes through interpolation a smooth, animated, time-encoded transition between them. A designer could choose one of the interpolated states for further development, or could use the interpolation as an iterative modeling technique to produce instances of the object as it transitions, i.e. *morphs* from one state to another (figures 2.24a-d).

A particularly interesting temporal modeling technique is *morphing*, in which dissimilar forms are blended to produce a range of hybrid forms that combine formal attributes of the "base" and



2.26.
The Ost/Kuttner Apartments (1996),
New York, USA, architect
Kolatan and Mac Donald.

2.27.
Ost/Kuttner Apartments: the
"cross-section referencing"
diagram.

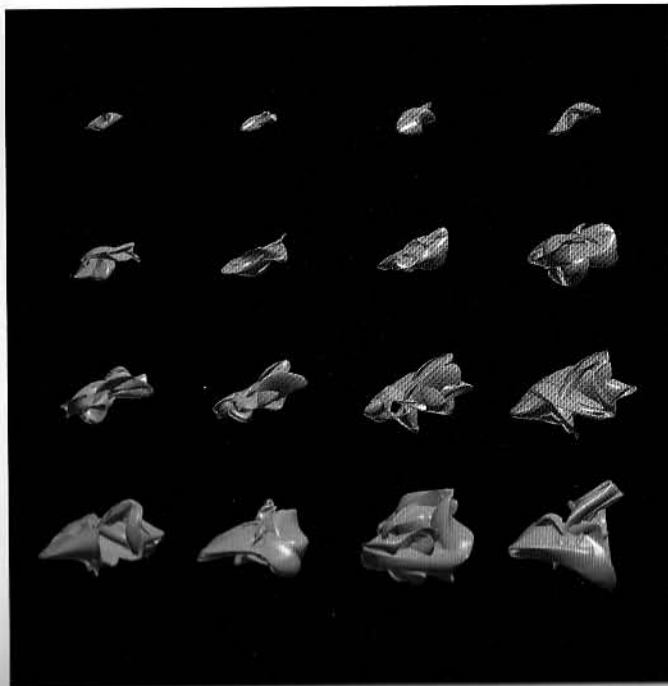


"target" objects. Kolatan and Mac Donald used morphing in a number of their projects. In *Housings*, a normative three-bedroom, two-and-a-half bathroom colonial house was used as a "base" object that was then morphed into a range of everyday objects as "targets," producing a large range of what they call "chimerical" designs (figures 2.25a–e). In the *Ost/Kuttner Apartments* (1996, figure 2.26), they digitally blended cross-referenced sectional profiles of common household furniture, such as a bed, sink, sofa, etc., to generate new hybrid forms that establish a "chimerical condition between furniture, space, and surface"²⁸ (figure 2.27). Kolatan and Mac Donald intentionally employed digital generative processes whose outcomes were "unknown and impossible to preconceive or predict,"²⁹ i.e. they relied on processes characterized by non-linearity, indeterminacy and emergence, which are discussed later in this chapter.

Other techniques for the metamorphic generation of form include deformations of the modeling space around an object using a bounding box (lattice deformation), a spline curve, or one of the coordinate system axes or planes, whereby an object's shape conforms to the changes in geometry of the modeling space. In *path* animation, for example, an object is deformed as it moves along a selected path (figure 2.28).

with animation: four
parallel volumes
move along four
separate curved paths.

29.
Interactivator (1995)
John and Julia
Frazer: experimental
evolution of form by
interaction with actual
visitors and
environmental sensors
reprogramming by
Manit Rastogi, Patrick
Gibson and Peter
Graham).

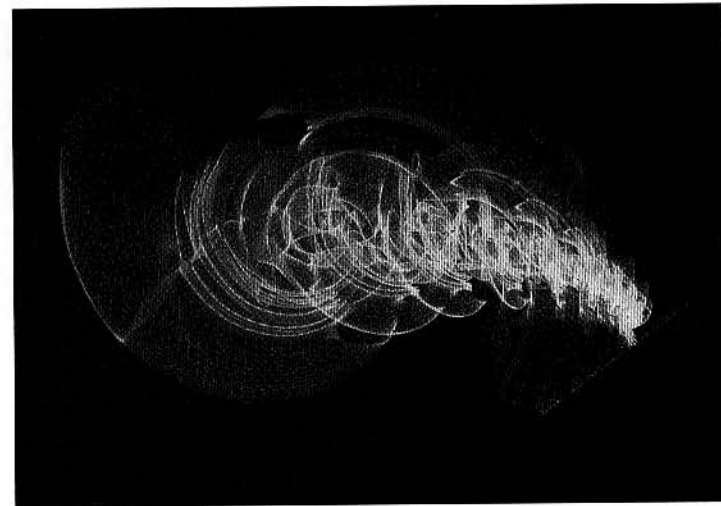


GENETICS

The "rules" that direct the genesis of living organisms, that generate their form, are encoded in the strands of DNA. Variation within the same species is achieved through gene crossover and mutation, i.e. through the iterative exchange and change of information that governs the biological morphogenesis.

The concepts of biological growth and form, i.e. the evolutionary model of nature, can be applied as the generative process for architectural form as well, argues John Frazer in his book *Evolutionary Architecture*.³⁰ According to Frazer, architectural concepts are expressed as a set of generative rules, and their evolution and development can be digitally encoded. The generative script of instructions produces a large number of "prototypical forms" which are then evaluated on the basis of their performance in a simulated environment.³¹ According to Frazer, the emergent forms are often unexpected.

The key concept behind the evolutionary approach to architecture is that of the genetic algorithm, "a class of highly parallel evolutionary, adaptive search procedures," as defined by Frazer. Their key characteristic is a "a string-like structure equivalent to the chromosomes of nature," to which the rules of reproduction, gene crossover and mutation are applied. Various parameters are encoded into "a string-like structure" and their



2.30.
X Phylum project by
Karl Chu.



2.31.
The FEM analysis for the fabric envelope of "Dynaform," BMW Pavilion at the IAA'01 Auto Show in Frankfurt, Germany, architects Bernhard Franken and ABB Architekten.

values changed, often randomly, during the generative process. A number of similar forms, "pseudo-organisms," are generated (figure 2.29), which are then selected from the generated populations based on a predefined "fitness" criteria. The selected "organisms," and the corresponding parameter values, are then crossbred, with the accompanying "gene crossovers" and "mutations" thus passing "beneficial and survival-enhancing traits" to new generations. Optimum solutions are obtained by small incremental changes over several generations.

Karl Chu's approach to digital morphogenesis and to what he calls the "proto-bionic" architecture is a formal system based on the generative logic of the *Lindermayer System* (L-System)³² and its implementation in digital modeling software, where it is used for the simulation of plant growth. L-systems are based on a recursive, rule-based branching system, conceived on the simple technique of rewriting, in which complex objects are created by successively replacing parts of an initially constructed object using a set of simple rewriting rules. The generative rules of an L-system can be very succinctly expressed. A simple set of carefully defined rules can produce a very complex object in a recursive process consisting of only a few levels (figure 2.30).

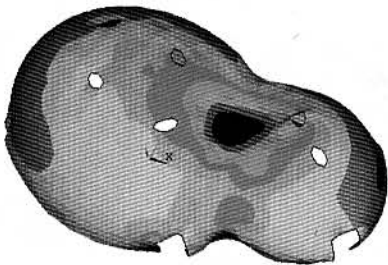
In both approaches to generative design based on biological metaphors, the task of the architect is to define the common

source of form, the "genetic coding" for a large family of similar objects, in which variety is achieved through different processes of "reproduction." As was the case with other contemporary approaches to design, in processes of genetic coding the emphasis shifts to articulating the inner logic of the project rather than the external form.

PERFORMATIVE ARCHITECTURE

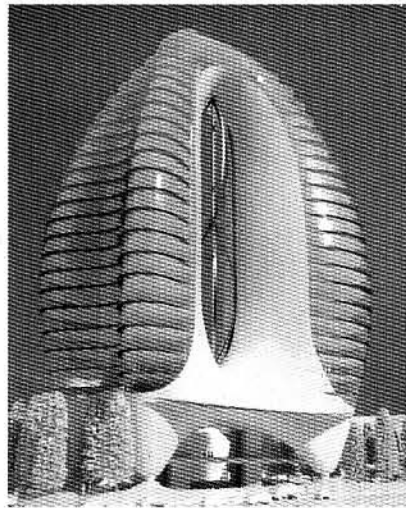
Another kind of architecture is also emerging, using building performance as a guiding design principle and adopting a new list of performance-based priorities for the design of cities, buildings, landscapes and infrastructures. This new kind of architecture places broadly defined performance above form-making; it utilizes the digital technologies of quantitative and qualitative performance-based simulation to offer a comprehensive new approach to the design of the built environment.

In this new information- and simulation-driven design context, the emerging paradigm of performance-based design is understood very broadly – its meaning spans multiple realms, from financial (the owner's perspective), spatial, social and cultural to purely technical (structural, thermal, acoustical, etc.). The emphasis on building performance (again, broadly understood from the financial, spatial, social, cultural, ecological and technical perspective) is redefining expectations of the building design, its processes and practices.

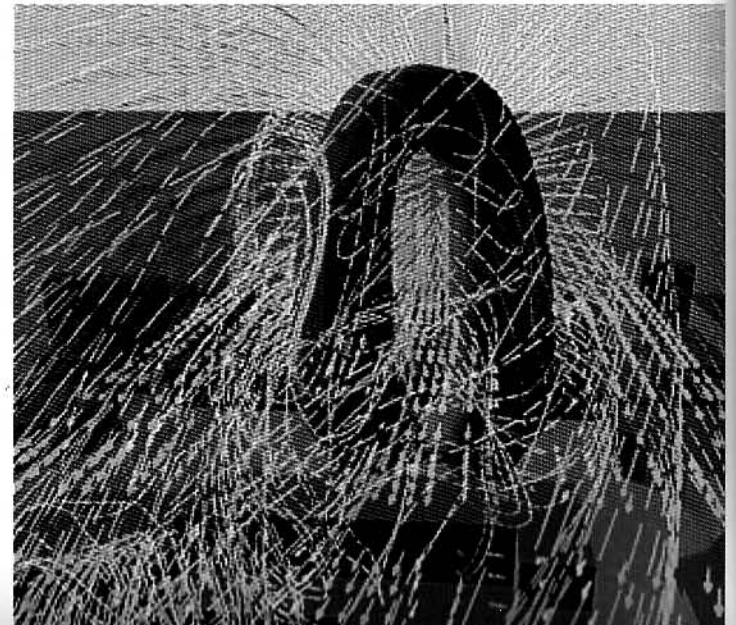


2.32.
The FEM analysis for the glass envelope of the "Bubble," BMW Pavilion at the IAA'99 Auto Show in Frankfurt, Germany, architects Bernhard Franken and

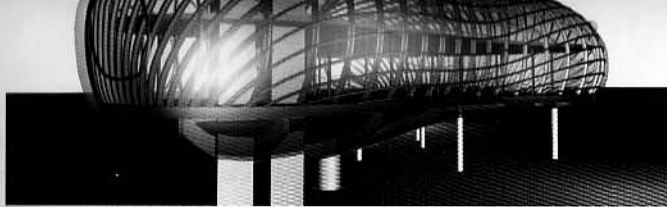
2.33.
Project ZED (1995), London, UK, architect Future Systems.



2.34.
The CFD analysis of wind flows for *Project ZED*



2.30.
Kunsthau (2003),
Graz, Austria,
architects Peter Cook
and Colin Fournier.



Analytical computational techniques based on the finite-element method (FEM), in which the geometric model is divided into small, interconnected mesh elements, are used to accurately perform structural, energy and fluid dynamics analyses for buildings of any formal complexity. These quantitative evaluations of specific design propositions can be qualitatively assessed today thanks to improvements in graphic output and visualization techniques (figures 2.31 and 2.32). By superposing various analytical evaluations, design alternatives could be compared with relative simplicity to select a solution that offers optimal performance.

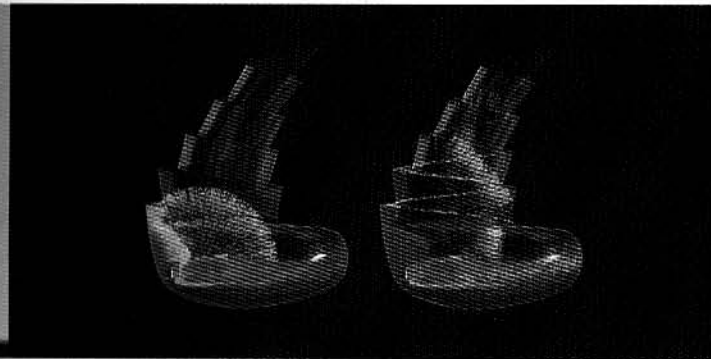
In computational fluid dynamics (CFD) software, used mainly to analyze airflows within and around buildings, fluid flow physics are applied to the digital model of a building to compute not only the dynamic behavior of the fluids (air, smoke, water, etc.), but also the transfer of heat mass, phase change (such as the freezing of water), chemical reactions (such as combustion), and stress or deformation of building structure (in fire, etc.).

Future Systems, a design firm from London, used CFD analysis in a particularly interesting fashion in its *Project ZED*, the design of a multiple-use building in London (1995, figure 2.33). The building was meant to be self-sufficient in terms of its energy needs by incorporating photovoltaic cells in the louvers and a giant wind turbine placed in a huge hole in its center. The curved form of the façade was thus designed to minimize the impact of the wind at the building's perimeter and to channel it towards the turbine at the center. The CFD analysis was essential in determining the optimal performance of the building envelope (figure 2.34).

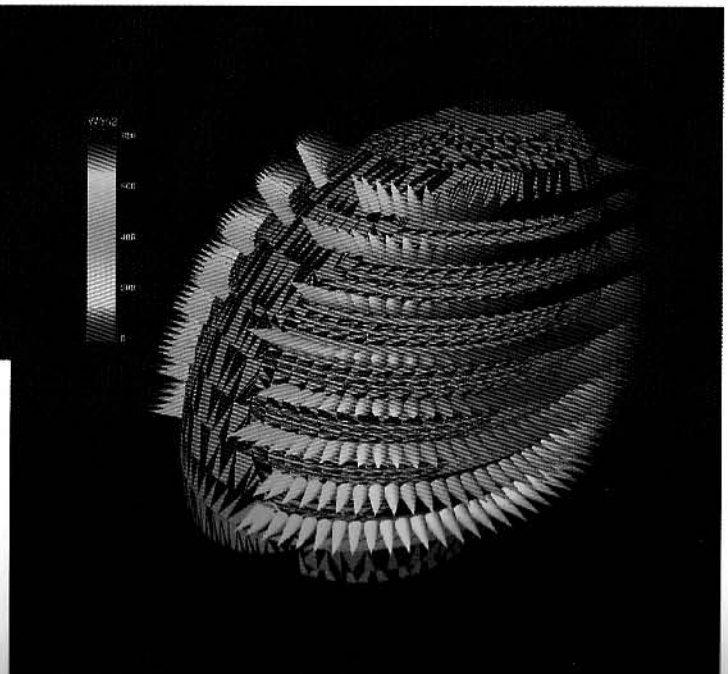
The original blobby shape of Peter Cook's and Colin Fournier's competition winning entry for the *Kunsthau in Graz, Austria* (2003, figure 2.35), was altered somewhat after the digital structural analysis, by consulting engineers Bollinger + Grohmann from Frankfurt, revealed that its structural performance could be improved with minor adjustments in the overall form. Likewise, Foster and Partners' design for the main chamber of the *Greater London Authority (GLA) Headquarters* (2002, figure 2.36) had to undergo several significant changes after engineers from Arup analyzed its acoustical performance using in-house developed acoustic wave propagation simulation software (figure 2.37). It is interesting to note that the "pebble"-like form of the building resulted from optimizing its energy performance by minimizing the surface area



2.36.
GLA Headquarters
(2002), London, UK,
architect Foster and
Partners.



2.37.
GLA Headquarters:
one of the acoustical
studies (by Arup).



2.38.
GLA Headquarters:
one of the solar studies
(by Arup).

exposed to direct sunlight (figure 2.38). The building's "blobby" form is actually a deformed sphere, which has a 25% smaller surface area than a cube of identical volume, resulting in a reduced solar heat gain and heat loss through the building's skin. The cladding configuration was a direct outcome of the analysis of sunlight patterns throughout the year.

Foster's performative approach to the design of the GLA building could imply a significant shift in how "blobby" forms are perceived. The sinuous, highly curvilinear forms could become not only an expression of new aesthetics, or a particular cultural and socio-economic moment born out of the digital revolution, but also an optimal formal expression for the new ecological consciousness that calls for sustainable building. If wind turbines were to become a reality of mankind's future, as futuristic designs by Future Systems suggest, the built environment would attain new morphology in which "boxes" could become as exotic as "blobs" are today.

Although digital technologies, in particular performance-based simulations, have made the notion of performative architecture possible, challenges and opportunities do exist in the ways these technologies are being conceptualized and used. Instead of being used in a passive, "after-the-fact" fashion, i.e. after the building form has been articulated, as is currently the case, analytical computation could be used to actively shape the buildings in a dynamic fashion, in a way similar to how animation software is used in contemporary architecture. An already-structured building topology, with a generic form, could be subjected to dynamic, metamorphic transformation, resulting from the computation of performance targets set at the outset. This dynamic range of performative possibilities would contain, at one end, an unoptimized solution and, at the other end, an optimized condition (if it is computable), which might not be an acceptable proposition from an aesthetic, or some other, point of view. In that case, a sub-optimal solution could be selected from the in-between performative range, which could potentially satisfy other non-quantifiable performative criteria.

This new kind of analytical software would preserve the topology of the proposed schematic design but would alter the geometry in response to optimizing a particular performance criteria (acoustic, thermal, etc.). For example, if there is a particular geometric configuration comprised of polygonal surfaces, the number of faces, edges, and vertices would remain unchanged (i.e. the topology does not change), but the shapes

(i.e. the geometry) would be adjusted (and some limits could be imposed in certain areas). The process of change could be animated, i.e. from the given condition to the optimal condition, with the assumption that the designer could find one of the in-between conditions interesting and worth pursuing, even though it may not be the most optimal solution.

NON-LINEARITY, INDETERMINACY AND EMERGENCE

Contemporary approaches to architectural design have abandoned the determinism of traditional design practices and have embraced the directed, *precise indeterminacy* of new digital processes of conception. Instead of working on a *parti*, the designer constructs a generative system of formal production, controls its behavior over time, and selects forms that emerge from its operation. In this model of design, a system of influences, relations, constrains or rules is defined first through the processes of *in-formation*, and its temporal behavior is specified; the resulting structure of interdependences is often given some generic form (*formation*), which is then subjected to the processes of *de-formation* or *trans-formation*, driven by those very same relations, influences or rules imbedded within the system itself.

The new approaches to design open up a formal universe in which essentially curvilinear forms are not stable but may undergo variations, giving rise to new possibilities, i.e. the *emergent form*. The formal complexity is often intentionally sought out, and this morphological intentionality is what motivates the processes of construction, operation and selection. The designer essentially becomes an "editor" of the morphogenetic potentiality of the designed system, where the choice of emergent forms is driven largely by the designer's aesthetic and plastic sensibilities. The capacity of digital, computational architectures to generate "new" designs is, therefore, highly dependent on the designer's perceptual and cognitive abilities, as continuous, dynamic processes ground the emergent form, i.e. its discovery, in qualitative cognition. Even though the technological context of design is thoroughly externalized, its arresting capacity remains internalized. The generative role of new digital techniques is accomplished through the designer's simultaneous interpretation and manipulation of a computational construct (topological surface, isomorphic field, kinetic skeleton, field of forces, parametric model, genetic algorithm, etc.) in a complex discourse that is continuously reconstituting itself – a "self-reflexive" discourse in which graphics actively shape the designer's thinking process. For instance,

designers can see forms as a result of reactions to a context of "forces" or actions, as demonstrated by Greg Lynn's work. There is, however, nothing automatic or deterministic in the definition of actions and reactions; they implicitly create "fields of indetermination" from which unexpected and genuinely new forms might emerge; unpredictable variations are generated from the built multiplicities.

It is precisely the ability of "finding a form" through dynamic, highly non-linear, indeterministic systems of organization that gives digital media a critical, generative capacity in design. Non-linear systems change indeterminately, continually producing new, unexpected outcomes. Their behavior over time cannot be explained through an understanding of their constituent parts, because it is the complex web of interdependencies and interactions that define their operation. In addition, in non-linear systems, it is often the addition or subtraction of a particular kind of information that can dramatically affect its behavior – in other words, a small quantitative change can produce a disproportionately large qualitative effect. It is this inherent capacity for "threshold" behavior that assigns to non-linearity the qualities of emergent behavior and infinite potential for change.

By openly embracing non-linearity, indeterminacy and emergence, the new digital design techniques challenge conventions such as stable design conceptualization, monotonic reasoning and first order logic that were (and still are) the underlying foundation for the design of mainstream computational tools for architectural production. In contemporary computational approaches to design, there is an explicit recognition that admittance of the unpredictable and unexpected is what often paves the way to poetic invention and creative transformation. The non-linearity, indeterminacy and emergence are intentionally sought out.

IT IS NOT ABOUT "BLOBS"

The changes brought about by the Information Age and globalization, as its most radical manifestation, are having a dramatic and profound impact on societies, economies and cultures worldwide. Architects, as they have done for centuries, are trying to interpret these changes and find an appropriate expression for an architecture that captures the zeitgeist of the dawn of the Information Age, which befits the information revolution and its effects. There is a wide range of approaches, discussed in this chapter, all of which express the unprecedented generative potentiality of digital techniques. The "blobby" aesthetics, which seem to be pervasive in the projects of the avant-garde, are often sidetracking the critical discourse into the more immediate territory of formal expression and away from more fundamental possibilities that are opening up, such as the opportunity for architects to reclaim the lost ground and once again become fully engaged in the act of building (as *information* master-builders). This is not to say that the profession should not maintain a critical attitude towards the potentiality of the digital, but that it should attempt to see beyond the issues of the formal aesthetics. Some extravagant claims were made, of course, and some unreasonable expectations were projected, which is not surprising given the totalizing fashion with which the digital domain is embraced in certain academic circles. But speculative design work, enabled by digital technologies, should at least provoke a healthy debate about the possibilities and challenges of the digital future.

Obviously, the "blobs" will not have a significant impact on architecture's future if they are understood in formal terms alone, or if they are seen as utopian architectural visions, as already happened in the 1960s. The challenge for the profession is to understand the appearance of the digitally-driven generative design and production technologies in a more fundamental way than as just tools for producing "blobby" forms.

NOTES

- 1** The Möbius strip, named after August Möbius, the German mathematician who first published this single-sided figure in 1865, can be simply constructed by connecting two ends of a twisted linear strip.
- 2** The Klein bottle is an edgeless, self-intersecting surface.
- 3** Le Corbusier. *Towards a New Architecture* (translated by F. Etchells). New York: Dover Publications, 1931.
- 4** Roberto Bonola. *Non-Euclidean geometry: A Critical and Historical Study of its Development* (translated by H.S. Carslaw). New York: Dover Publications, 1955.
- 5** *Ibid.*
- 6** It is important to note, however, that unequally weighted control points become necessary for constructing the curves of conic sections: circles, ellipses, parabolas, and so on.
- 7** Mathematically, this means that the curve is continuous but has no derivative at the cusp.
- 8** The first derivative is continuous, but the second one is not.
- 9** Both the first and second derivatives are continuous.
- 10** For more information about NURBS parametric definition, see Les Piegl and Wayne Tiller, *The NURBS Book*, 2nd edition. New York: Springer, 1997.
- 11** Mark Burry. "Paramorph" in Stephen Perrella (ed.), *AD Profile 141: Hypersurface Architecture II*. London: Academy Editions, 1999.
- 12** *Ibid.*
- 13** Marcos Novak. "Transarchitectures and Hypersurfaces" in Stephen Perrella (ed.), *AD Profile 133: Hypersurface Architecture*. London: Academy Editions, 1998.
- 14** *Ibid.*
- 15** Greg Lynn. *Animate Form*. Princeton: Princeton Architectural Press, 1998.
- 16** *Ibid.*
- 17** *Ibid.*
- 18** *Ibid.*
- 19** Ignasi de Sola Morales. *Differences: Topographies of Contemporary Architecture*. Cambridge: MIT Press, 1997.
- 20** Lynn, *Animate Form*. *op cit.*
- 21** *Ibid.*
- 22** D'Arcy Thompson. *On Growth and Form*. Cambridge (UK): Cambridge University Press, 1917.
- 23** Lynn. *Animate Form*. *op cit.*
- 24** *Ibid.*
- 25** *Ibid.*
- 26** Nicholas Negroponte. *The Architecture Machine*. Cambridge: MIT Press, 1970.
- 27** MVRDV. *Metacity/Datatown*. Rotterdam, the Netherlands: 010 Publishers, 1999.
- 28** Sulan Kolatan. "More Than One/Less Than Two_RESIDENCE[IS]" in Peter C. Schmal (ed.), *Digital, Real: Blobmeister: First Built Projects*. Basel: Birkhauser, 2001. pp. 68–79.
- 29** *Ibid.*
- 30** John Frazer. *Evolutionary Architecture*. London: Architectural Association, 1995.
- 31** *Ibid.*
- 32** A mathematical theory of plant development named after its inventor, biologist Aristid Lindenmayer (1925–89).